wØgvwÎK MwZ

f~wgKv

c‡eKvi BDwb‡U Avgiv MwZ msµvš— iwkgvjv, h_v- miY, †eM, ZiY Ges Ae¯'vb †f±i wb‡q Av‡jvPbv K‡iwQ| GQvov Av‡jvPbv K‡iwQ cm½ we›`yI cm½ KvVv‡gv m¤ú‡K| Gici GKgvwÎK Z_v mij ^iwLK MwZ eYbvq Avgiv Zv e¨envi K‡iwQ| GKgvwÎK MwZwe`¨vq †jLwPÎ, cišyev¯—e mgxKiY Ges Av‡cw¶K †e‡Mi avi Yv m¤úó Kiv n‡q‡Q| Avgiv BwZg‡a¨ Av‡jvPbv K‡iwQ GKgvwÎK cm½ KvVv‡gvi g‡Zv wØgvwÎK Ges wÎgvwÎK cm½ KvVv‡gv‡Z e¯'Ki Yvi miY m¤ú‡K|

G Aa¨v‡q Avgiv wØgvwÎK cm½ KvVv‡gv‡Z Z_v mgZ‡j MwZkxj e¯'i †¶‡Î H mKj iwkgvjvi AeZviYv Kie| wØgvwÎK MwZ‡Z †eM Ges Zi‡Yi †f±i i|c fvjfv‡e Dcjwä Kiv hvq| GB MwZ‡Z e¯'KYv GKB †iLv ev A¶ eivei MwZkxj n‡e Ggb †Kvb K_v bvB| eis `yB A‡¶i mv‡c‡¶ ev mgZ‡ji Dci e¯'KYvi MwZkxjZv‡KB GLv‡b Av‡jvPbv Kiv n‡e| GKgvwÎK MwZi †¶‡Î j× iwkgvjv mn‡RB wÎgvwÎK MwZi †¶‡Î m¤úmviZ Kiv hv‡e| G BDwb‡U Avgiv mgZjxq wØgvwÎK MwZ Av‡jvPbvq mxgve× _vKe| D`vniY wn‡m‡e cw¶ß e¯'i MwZ Ges e„Ëxq MwZi AvjvPbv Aš—f³ n‡e|

cvV - 1

MwZ welqK iwkgvjvi Ges MwZi mgxKiY mg‡ni wØgvwÎK †f±i iƒc

D‡Ïk¨

G cv‡Vi †k‡l Avcwb-

- wØgvwÎK cm½ KvVv‡gv‡Z †Kvb we›`yi Ae¯'vb †f±i wbYq Ki‡Z cvi‡eb,
- MwZmsµvšÍ wewfbœ iwk‡K c‡qvRb Abyhvqx wØgvwÎK KvVv‡gv‡Z †f±i iƒ‡c cKvk Ki‡Z cvi‡eb,
- wØgvwÎK MwZi †¶‡Î MwZi mgxKiY mg~‡ni †f±i iƒc cwZcv`b Ki‡Z cvi‡eb|

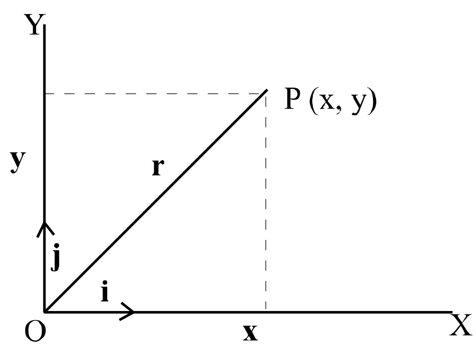
3.1.1 MwZ welqK iwkgvjvi †f±i cKvk

Avgiv ceeZx BDwb‡U Av‡jvPbv K‡iwQ †h, cm½ KvVv‡gvi gƒj we›`y mv‡c‡¶ †Kvb we›`yi Ae¯'vb‡K †h †f±i w`‡q eY©bv nq Zv‡K Ae¯'vb †f±i e‡j [ Abyˆ”Q` 1.1]| Ae¯'vb †f±i‡K **r** Øviv cKvk Kiv nq| wØgvwÎK †¶‡Î x A¶ I y A‡¶i mv‡c‡¶ GKwU we›`yi ¯'vbv¼ wPýZ nq `ywU iwk w`‡q| GKwU x A¶ †_‡K we›`yi `~iZ, AbÿwU y-A¶ †_‡K we›`yi `~iZ| Avgiv wjwL P we›`yi ¯'vbv¼ (x, y) ev ms‡¶‡c p(x,y)| x, I y A‡¶i †Q` we›`y‡K gƒj we›`y(0, 0) aiv nq| gƒj we›`y mv‡c‡¶ P we›`yi Ae¯'vb †f±i

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \quad \ldots \quad (3.1)$$

GLv‡b **i**, **j** h_vµ‡g x I y A¶ Awfgyx GKK †f±i|

wØZxq BDwb‡U Avgiv t ebvg x †jL wPÎ Av‡jvPbv K‡iwQ| †mLv‡b MwZkxj e¯'zmg‡qi mv‡c‡¶ GKB †iLv eivei MwZkxj †_‡K‡Q| G BDwb‡U NUbv wKQyUv Avjv`vfv‡e Dc¯'vcb Kiv n‡e| G‡¶‡Î †iLvi cwie‡Z e¯'KYvi ¯'vbvšÍi n‡e xy mgZ‡j| e¯'z MwZc_ †`Lvb n‡e wØgvwÎK KvVv‡gv‡Z| †Kvb e¯'z MwZc_‡K A‡bK mgq Avgiv c_‡iLv (Trajectory) e‡j AwfwnZ Kie|



wPÎÑ 3.1

3.2 bs wP‡Î GKwU e¯'z c_g Ae¯'vb $\mathbf{r_i}$ †_‡K Ae¯'vb $\mathbf{r_f}$ Ges e¯'z miY $\Delta\mathbf{r}$ †`Lv‡bv nj|
mi‡Yi w`K Aw` Ae¯'vb †_‡K †kl Ae¯'v‡bi w`‡K|
c_g Ae¯'v‡b e¯'z Ae¯'vb †f±i $\mathbf{r_i}$ Ges †kl Ae¯'v‡b e¯'z Ae¯'vb †f±i $\mathbf{r_f}$ |

GLv‡b $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$

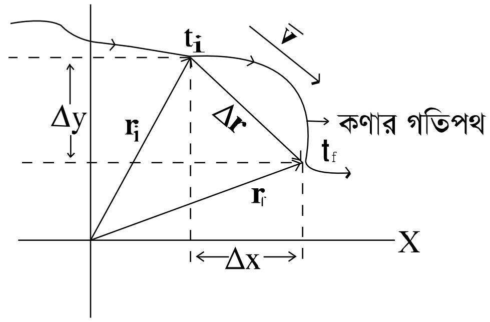
$= (x_f\,\mathbf{i} + y_f\,\mathbf{j}) - (x_i\,\mathbf{i} + y_i\,\mathbf{j})$

$= (x_f - x_i)\mathbf{i} + (y_f - y_i)\mathbf{j}$

ev, $\Delta \mathbf{r} = \Delta x\mathbf{i} + \Delta y\mathbf{j}$ … … … … … … … .. … (3.2)

GLv‡b $\Delta x = x_f - x_i$ Ges $\Delta y = y_f - y_i$

h_vµ‡g x Ges y A¶ eivei e¯'i mi‡Yi Dcvsk|



wPÎ 3.2

†eM

†h †Kvb mg‡qi e¨eav‡b †Kvb e¯'i M‡o GKK mg‡q †h miY nq Zv‡K e¯'i Mo †eM e‡j| Δt mg‡qi e¨eav‡b †Kvb e¯' miY $\Delta \mathbf{r}$ n‡j Mo †eM

$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$ (Mo eY©bv‡bvi Rb¨ †Kvb cZ‡Ki Dci ô—ô evi wPý e¨eüZ nq)

ev, $\bar{\mathbf{v}} = \frac{\Delta (x\mathbf{i}+y\mathbf{j})}{\Delta t}$

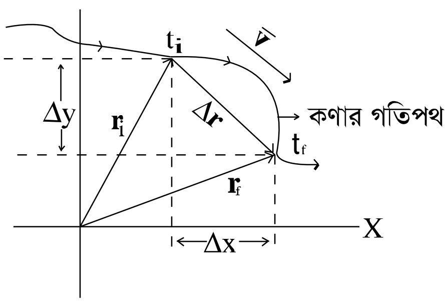
ev, $\bar{\mathbf{v}} = \frac{\Delta x\mathbf{i}+\Delta y\mathbf{j}}{\Delta t}$

ev, $\bar{\mathbf{v}} = \frac{\Delta x}{\Delta t}\mathbf{i} + \frac{\Delta y}{\Delta t}\mathbf{j} = \bar{V}_x\mathbf{i} + \bar{V}_y\mathbf{j}$

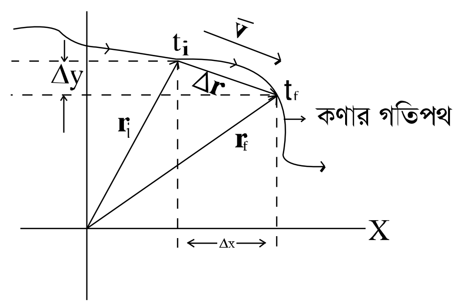
GB Mo †e‡Mi w`K $\Delta \mathbf{r}$ Gi w`‡K| wPÎ (3.2)

mg‡qi e¨eavb k‡b¨i mgxceZx n‡j Mo †e‡Mi mxgvšK Ae¯'vi (in the limit) gvb‡K Zvr¶wYK †eM ev †eM e‡j,

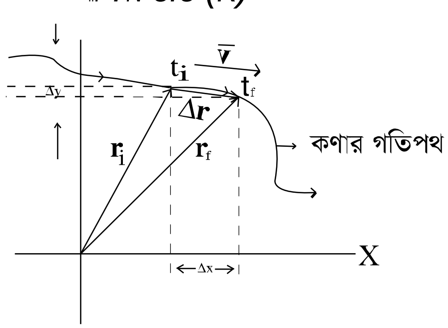
$$\mathbf{V} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \quad \text{… … … … … … … .. … (3.3)}$$



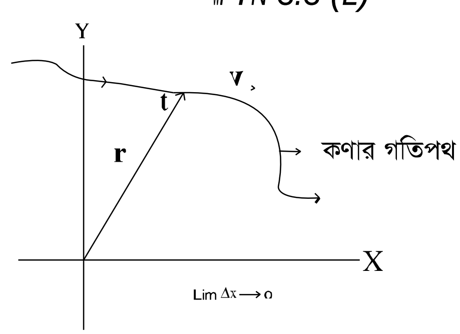
wPÎ 3.3 (K)



wPÎ 3.3 (L)



wPÎ 3.3 (M)



wPÎ 3.3 (N)

wPÎ 3.3 Gi K, L I M-G †`Lvb n‡q‡Q †Kvb e¯‘ MwZi †¶‡Î mg‡qi e¨eavb $\Delta t$ µgk K‡g hv‡”Q| GLv‡b e¯‘i MwZc‡_i †jL wP‡Îi Dci GKwU mxgvšÍK cwigvY eySv‡bvi †Póv Kiv n‡q‡Q| K †_‡K M wP‡Î ¶z`ª †_‡K ¶z`ªZi mgq e¨eavb †`Lv‡bv n‡”Q| mg‡qi e¨eavb wnmv‡ei myweavi Rb¨ t_i ‡K w¯’i †i‡L t_f ‡K t_i Gi mgxceZx Kiv n‡q‡Q | mxgvšÍK gv‡bi ‡¶‡Î $\Delta \mathbf{r}$ MwZc‡_i ¯úk‡Ki mgvšÍivj nq| Gi AwfgyL nq MwZi w`‡K| wKšÍ hLb $t_i$ Ges $t_f$ Gi cv_©K¨ Lye Kg nq| A_©vr $\Delta t \to 0$ nq, ZLb MwZ c‡_i †Kvb we›`y‡Z †eM $\mathbf{v}$ Gi w`K nq H we›`y‡Z AswKZ ¯úkK eivei (wPÎ 3.3 N)|

mgxKiY 3.3 †_‡K †`Lv hv‡”Q †eM $\mathbf{v}$ n‡”Q mg‡qi mv‡c‡¶ Ae¯’vb †f±‡ii cwieZ©‡bi nvi| †h‡nZz

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \frac{d}{dt}(x\mathbf{i} + y\mathbf{j})$$

$$= \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j}$$

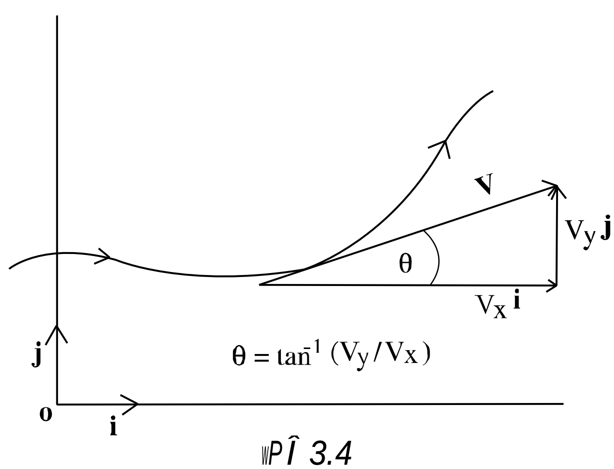
wKšÍ $\frac{dx}{dt}$ n‡”Q mg‡qi mv‡c‡¶ x ¯’vbvs‡Ki cwieZ©‡bi nvi hv x- A¶ eivei e¯‘i †e‡Mi Dcvsk v_x | Abyiƒc fv‡e $\frac{dy}{dt}$ n‡e y A¶ eivei e¯‘i †e‡Mi Dcvsk v_y |

$$\therefore \mathbf{v} = v_x\,\mathbf{i} + v_y\,\mathbf{j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

†e‡Mi gvb A_©vr `ª“wZ $\mathbf{v}$ n‡j

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

MwZkxj e¯‘wUi †h †Kvb gyn~‡Z©i †e‡Mi w`K †eM †f±i $\mathbf{v}$ I x- A‡¶i AšÍfz©³ †KvY θ Øviv eY©bv Kiv hvq|



wPÎ 3.4

wPÎ 3.4 †_‡K †`Lv hvq, $\tan\theta = \frac{v_y}{v_x}$

$$\text{ev}\quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

ZiY

†h †Kvb mgq e¨eav‡b †Kvb e¯'i M‡o cwZ GKK mg‡q †e‡Mi †h cwieZb nq ZvB Mo ZiY| Δt mg‡qi e¨eav‡b †Kvb MwZkxj e¯'i †e‡Mi cwieZb $\Delta \mathbf{v}$ n‡j Mo ZiY

$\bar{a} = \frac{\Delta \mathbf{v}}{\Delta t}$ GLv‡b Mo eySv‡Z $\mathbf{a}$-Gi Dci evi ô—ô wPý e¨eüZ n‡q‡Q|

$$\text{ev } \bar{a} = \frac{\Delta v_x}{\Delta t}\mathbf{i} + \frac{\Delta v_y}{\Delta t}\mathbf{j}$$

$$= \bar{a}_x \mathbf{i} + \bar{a}_y \mathbf{j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.7)$$

GB Mo Zi‡Yi w`K $\Delta \mathbf{v}$ Gi w`‡K| mgq e¨eavb k‡b¨i mgxceZx n‡j Mo Zi‡Yi mxgvšK gvb‡K Zvr¶wYK ZiY ejv nq| A_vr

Zvr¶wYK ZiY,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.8)$$

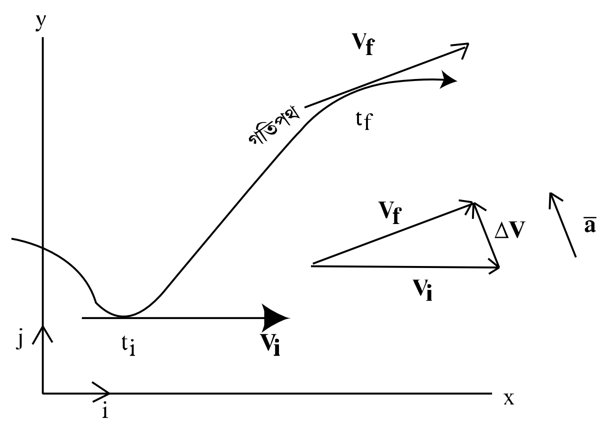
mgxKiY (3.8) †_‡K †`Lv hv‡"Q ZiY $\mathbf{a}$ n‡"Q mg‡qi mv‡c‡¶ †e‡Mi cwieZ‡bi nvi $\frac{d\mathbf{v}}{dt}$ |

†h‡nZz

$$\mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j}$$

$$\therefore \mathbf{a} = \frac{d}{dt}(v_x \mathbf{i} + v_y \mathbf{j})$$

$$= \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j}$$



wPÎ 3.5

wKšz $\frac{dv_x}{dt}$ n‡"Q mg‡qi mv‡c‡¶ e¯'i †e‡Mi x Dcvs‡ki cwieZ‡bi nvi A_vr x A¶ eivei Zi‡Yi Dcvsk $a_x = \frac{dv_x}{dt}$ |

Abyiƒc fv‡e y A¶ eivei Zi‡Yi Dcvsk $\frac{dv_y}{dt} = a_y$

$$\therefore \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.9)$$

3.1.2 MwZi mgxKi‡Yi †f±i i|c cwZcv`b

BDwbU -2 G Avgiv GKgwÎK cm½ KvVv‡gv‡Z MwZi mgxKiY mgn~ cwZcv`b K‡iwQ| GLb Avgiv mgZ‡j A\_vr wØ-gwÎK MwZi †¶‡Î MwZi mgxKiY¸‡jv cwZcv`b Kie| aiv hvK t mg‡q MwZkxj †Kvb e¯'i,

Ae¯'vb †fKUi, $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ … … … … … … .. …. … … … ..(3.9a)

†eM, $\mathbf{v} = v_x\,\mathbf{i} + v_y\,\mathbf{j}$ … … … … … … .. …. … …(3.9b)

ZiY, $\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j}$ … … … … … … .. …. … … (3.9c)

Av‡Mi gZB Avgiv mgZ‡Y MwZkxj e¯'i Rb¨ mgxKiY¸wj cwZcv`b Kie| A\_vr ZiY $\mathbf{a}$ = a"eK| aiv hvK, mgq MYbvi ïi"‡Z A\_vr hLb t = 0 ZLb e¯'wUi Avw` Ae¯'vb †f±i $\mathbf{r}_0$ Ges Avw` †eM $\mathbf{v}_0$ |

A_vr Dcvs‡ki mvnv‡h¨ wjL‡j

Avw` Ae¯'vb †f±i, $\mathbf{r}_0 = x_0\,\mathbf{i} + y_0\,\mathbf{j}$ … … … … … … .. …. … … (3.10a)

Avw` †eM, $\mathbf{v}_0 = v_{x0}\,\mathbf{i} + v_{y0}\,\mathbf{j}$ … … … … … … .. …. … … (3.10b)

Gevi mgZ‡Y MwZi †¶‡Î mgxKiY¸wj cwZcv`b Kiv hvK|

(K) $\mathbf{v} = v_0 + \mathbf{a}t$

GKgwÎK MwZi †¶‡Î Avgiv Rwb $v = u + at$

x A¶ eivei Avw` †eM v_{x0} Ges t mg‡q †eM v_x Ges ZiY a_x ai‡j GKgwÎK MwZi mgxKiYwU n‡e|

$$v_x = v_{x0} + a_x\,t$$

mgxKiYwU †f±i i|‡c wjL‡j n‡e,

$v_x\,\mathbf{i} = v_{x0}\,\mathbf{i} + a_x\,t\,\mathbf{i}$ [∵ x A¶ eivei GKK †f±i $\mathbf{i}$]

Abyi|cfv‡e y A¶ eivei MwZi Rb¨ mgxKiY n‡e,

$$v_y\,\mathbf{j} = v_{y0}\,\mathbf{j} + a_y\,t\,\mathbf{j}$$

mgxKiY `yBwU †hvM Ki‡j cvIqv hvq,

$$v_x\,\mathbf{i} + v_y\,\mathbf{j} = v_{x0}\,\mathbf{i} + a_x\,t\,\mathbf{i} + v_{y0}\,\mathbf{j} + a_y\,t\,\mathbf{j}$$

ev, $\mathbf{v} = (v_{x0}\,\mathbf{i} + v_{y0}\,\mathbf{j}) + (a_x\,\mathbf{i} + a_y\,\mathbf{j})t$

ev, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ … … … … … … … .. …. … … (3.11)

[mgxKiY 3.10b ‡_‡K $v_{x0}\,\mathbf{i} + v_{y0}\,\mathbf{j} = \mathbf{v}_0$

Ges 3.9c †_‡K $a_x\,\mathbf{i} + a_y\,\mathbf{j} = \mathbf{a}$ ewm‡q]

(L) $\mathbf{r} = \mathbf{r}_0 + \frac{1}{2}(\mathbf{v}_0 + \mathbf{v})\,t$

GKgwÎK MwZi †¶‡Î Avgiv Rwb Mo‡eM $\bar{v} = \frac{u+v}{2}$

Ges $S = \left(\frac{u+v}{2}\right) t$

x A¶ eivei $s = x - x_0 = \left(\frac{v_{x0}+v_x}{2}\right) t$

ev, $x = x_0 + (v_{x0} + v_x)\ \frac{t}{2}$

ev, $x = x_0 + \frac{1}{2}(v_{x0} + v_x)\ t$

GB mgxKiYwUi †f±i iƒc n‡e

$x\mathbf{i} = x_0\mathbf{i} + \frac{1}{2}(v_{x0} + v_x)\ t\,\mathbf{i}$

Abiƒcfv‡e y A¶ eivei MwZi mgxKiYwUi †f±i iƒc n‡e

$y\mathbf{j} = y_0\,\mathbf{j} + \frac{1}{2}(v_{y0} + v_y)\ t\,\mathbf{j}$

mgxKiY `yÕwU †hvM K‡i cvB

$x\mathbf{i} + y\mathbf{j} = x_0\,\mathbf{i} + \frac{1}{2}(v_{x0}+v_x)\ t\,\mathbf{i} + y_0\,\mathbf{j}\,) + \frac{1}{2}(v_{y0}+v_y)\ t\,\mathbf{j}$

ev, $\mathbf{r} = (x_0\,\mathbf{i} + y_0\,\mathbf{j}\,) + \frac{1}{2}\ [\,(v_{x0}\,\mathbf{i} + v_{y0}\,\mathbf{j}\ + (v_x\ \mathbf{i} + v_y\ \mathbf{j})]\ t$

ev, $\mathbf{r} = \mathbf{r_0} + \frac{1}{2}(\mathbf{v_0} + \mathbf{v})\ t$ … … … ... ... …. ... … (3.12)

(M) $\mathbf{r} = \mathbf{r_0} + \mathbf{v_0}\ t + \frac{1}{2}\ \mathbf{a}t^2$

GKgvÎK MwZi †¶‡Î Avgiv Rvwb $S = ut + \frac{1}{2}\ a\ t^2$

X A¶ eivei $S = x - x_0 = v_{x0}\ t + \frac{1}{2}\ a_x\ t^2$

ev, $x = x_0 + v_{x0}\ t + \frac{1}{2}\ a_x\ t^2$

GB mgxKi‡Yi †f±i iƒc nj

$x\ \mathbf{i} = x_0\ \mathbf{i} + v_{x0}\ t\,\mathbf{i} + \frac{1}{2}\ a_x\ t^2\ \mathbf{i}$

Abyiƒcfv‡e y A¶ eivei MwZi mgxKi‡Yi †f±i iƒc nj

$y\,\mathbf{j} = y_0\ \mathbf{j} + v_{y0}\ t\,\mathbf{j} + \frac{1}{2}\ a_y\ t^2\,\mathbf{j}$

GB mgxKiY `yÕwU †hvM K‡i cvB,

$x\ \mathbf{i} + y\,\mathbf{j} = x_0\ \mathbf{i} + v_{x0}\ t\,\mathbf{i} + \frac{1}{2}a_x\ t^2\,\mathbf{i} + y_0\ \mathbf{j} + v_{y0}\ t\,\mathbf{j} + \frac{1}{2}\ a_y\ t^2\,\mathbf{j}$

ev, $\mathbf{r} = (x_0\ \mathbf{i}\ + y_0\ \mathbf{j}) + (v_{x0}\ \mathbf{i}\ + v_{y0}\ \mathbf{j}\,)\ t + \frac{1}{2}(a_x\,\mathbf{i} + a_y\,\mathbf{j}\,)\ t^2$

ev, $\mathbf{r} = \mathbf{r_0}\ + \mathbf{v_0}\ t + \frac{1}{2}\ \mathbf{a}\,t^2$ … … … ... ... …. ... … (3.13)

mvi ms‡¶c

wØgvwÎK Mo‡eM, $\bar{\mathbf{v}} = \dfrac{\Delta \mathbf{r}}{\Delta t}$

wØgvwÎK Zvr¶wYK †eM, $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$

wØgvwÎK MoZiY, $\bar{\mathbf{a}} = \dfrac{\Delta \mathbf{v}}{\Delta t}$

wØgvwÎK Zvr¶wYK ZiY $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$

mgZi‡Y MwZi mgxKi‡Yi wØgvwÎK †f±i i|c

(i) $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}\,t$

(ii) $\mathbf{r} = \mathbf{r}_0 + \dfrac{1}{2}(\mathbf{v}_0 + \mathbf{v})\,t$

(iii) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0\,t + \dfrac{1}{2}\,\mathbf{a}t^2$

cv‡VvËi gj~vqb

mwVK DË‡ii cv‡k wUK wPý (√) w`b

1| †KvbwU Mo Zi‡Yi msÁv ?

(K) $\bar{\mathbf{a}} = \dfrac{d\mathbf{v}}{dt}$

(L) $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$

(M) $\bar{\mathbf{a}} = \dfrac{\Delta \mathbf{v}}{\Delta t}$

(N) $\mathbf{a} = \underset{\Delta t \to 0}{\mathrm{Lt}} \dfrac{\Delta \mathbf{v}}{\Delta t}$

2| †KvbwU Zvr¶wYK †e‡Mi msÁv ?

(K) $\mathbf{v} = \dfrac{\mathbf{s}}{t}$

(L) $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

(M) $\bar{\mathbf{v}} = \dfrac{d\mathbf{r}}{dt}$

(N) $\mathbf{v} = \underset{\Delta t \to 0}{\mathrm{Lt}} \dfrac{\Delta \mathbf{v}}{\Delta t}$

3| e¯‘i Ae¯vb †f±i ej‡Z †KvbwU eySvq ?

(K) $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$

(L) $\Delta \mathbf{r} = \Delta x\mathbf{i} + \Delta y\mathbf{j}$

(M) $\mathbf{S} = (v_x\,\mathbf{i} + v_y\,\mathbf{j})\,t$

(N) $\Delta \mathbf{S} = (a_x\,\mathbf{i} + a_y\,\mathbf{j})\,t^2$

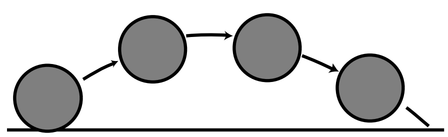
cvV -2
wbw¶ß e¯‘i cv‡mi MwZ I MwZc‡_i mgxKiY

G cv‡Vi †k‡l Avcwb

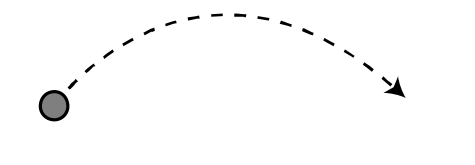
- *cvm wK Zv ej‡Z cvi‡eb,*
- *cv‡mi MwZ wØgvwÎK Zv e¨vL¨v Ki‡Z cvi‡eb,*
- *cvm ev wbw¶ß e¯‘i wØgvwÎK MwZ wbavi‡Yi mgxKiY cwZcv`b Ki‡Z cvi‡eb,*
- *cv‡mi mewaK D"PZvq I‡Vi mgq Kvj, mewaK D"PZv, DÇqb Kvj I Avbyf~wgK পাল্লা wbYq Ki‡Z cvi‡eb|*

3.2.1 wbw¶ß e¯‘ cvm

†Kvb e¯‘‡K Avbyf~wg‡Ki mv‡_ wZhK fv‡e evZv‡m Qy‡o w`‡j Zvi Dci AwfKl© ZiY KvR K‡i Ges mgq›`‡bi cfv‡e e¯‘wU µgk Dc‡i DV‡Z DV‡Z †eM k~b¨ nq| cieZ©x‡Z e¯‘wU GKB fv‡e f~c„‡ô wd‡i Av‡m| Avgiv dzUe‡j jvw_ gvivi `„k¨ i m‡½ mevB cwiwPZ| GKwU wXj evZv‡m Qy‡o †`qvi `„k¨ I Avgiv †`‡LwQ| dzUej ev wXj n‡jv| cv‡mi D`vniY|



*wPÎ 3.6 (K) dzUe‡ji MwZc_*



*wPÎ 3.6 (L) Qy‡o †`Iqv e‡ji MwZc_*

e¯‘ZxhKfv‡e Qy‡o w`‡j Zv AwaeË©vKvi c‡\_ Avevi f~c„‡ô wd‡i Av‡m| ZxhKfv‡e wbw¶ß †h †Kvb e¯‘i MwZ‡K Avgiv cvm ewj| evZv‡m wbw¶ß e¯‘i MwZi mgxKiY wbavi‡Yi †¶‡Î evZv‡mi evav‡K D‡c¶v Kiv nq| cv‡mi MwZ wØgvwÎK| ZxhKfv‡e wbw¶ß wXj, ey‡jU, ej BZ¨vw`i MwZ cvm MwZi D`vniY|

cvm MwZi †¶‡Î KwZcq iwki msÁv

cv‡mi MwZ msµvšÍ Av‡jvPbvq K‡qKwU iwki aviYv ¸i"Zc~Y©, Zv n‡jv:

(1) wb‡¶cb ‡eM (Velocity of projection) *t †h Aw` †e‡M cvm‡K k~‡b¨ ev evZv‡m wb‡¶cb Kiv nq Zv‡K wb‡¶cb †eM e‡j|*

(2) wb‡¶cb †KvY (Angle of projection) *t wb‡¶‡ci gyn~‡Z© wb‡¶cb †e‡Mi Awfgy‡L Abyf~wg‡Ki mv‡_ †h †KvY m„wó K‡i Zv‡K wb‡¶cb †KvY e‡j|*

(3) wb‡¶cb we›`y* (Point of projection) *t †h we›`y †_‡K cvmwU wbw¶ß nq Zv‡K wb‡¶cb we›`y e‡j|

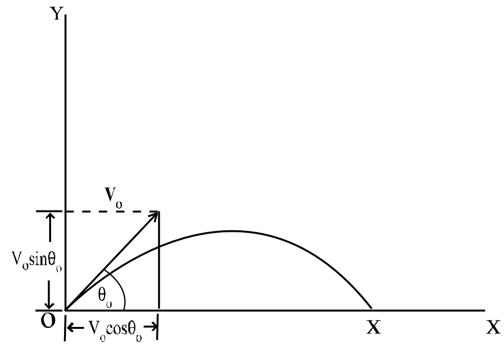
*(4) wePiY c_* (Trajectory) *t cv‡mi MwZc_‡K ev c_ †iLv‡K Gi wePiY c_ e‡j|*

(5) wePiY Kvj (Time of flight) *t Dr‡¶cb gyn~Z© †_‡K †h mgq ci cvm mgZ‡j wd‡i Av‡m Zv‡K wePiY Kvj e‡j|*

(6) পাল্লা (Range) *t cm½ mgZ‡ji †h we›`y‡Z cvm cwZZ nq Zv‡K cZb we›`y e‡j| wb‡¶cb we›`yI cZb we›`y gv‡eZx mij ˆiwLK `~iZ‡K cv‡mi পাল্লা e‡j|*

3.2.2 cv‡mi MwZ c‡_i mgxKiY

g‡bKwi, †h we›`y‡_‡K cvmwU wb‡¶c Kiv nj †mwU GKwU wØgvwÎK cm½ KvVv‡gvi gj– we›`y cm½ KvVv‡gvi abvZK Y A¶ উল্লম্ব eivei Lvov Dc‡ii w`‡K Ges abvZK X A¶ e¯‘wUi †h w`‡K Abyf~wgK miY nq †m w`‡K aiv nj| myZivs e¯‘ Aw` Ae¯vb ev cv_wgK Ae¯vb †f±i Gi x I y ¯vbvsK h_vµ‡g $x_0 = 0$ Ges $y_0 = 0$| e¯‘wUi Dci AwfKlR ZiY g Lvov wb‡Pi w`‡K KvR Ki‡e| myZivs G†¶‡Î e¯‘wUi ZiY n‡e



wPÎÑ 3.7

$\mathbf{a} = -g$ [GLv‡b $g = 9.8\ ms^{-2}$]|

Aveviv, $\mathbf{a} = a_x\,\mathbf{i} + a_y\,\mathbf{j}$

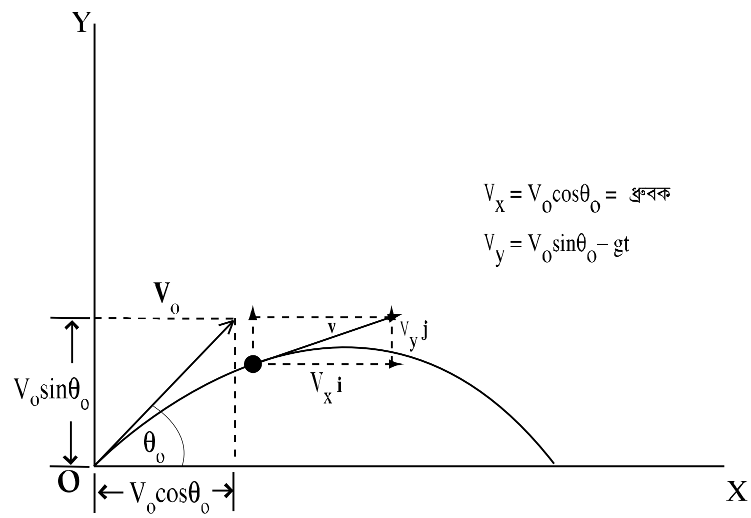
myZivs $a_x = 0$ Ges $a_y = -g$

Av‡iv aiv hvK, hLb e¯‘wU hvÎv ïi“ K‡i ZLb †eM wQj v_0 Ges †mB mg‡q A_vr $t = 0$ mg‡q Gi †eM $\mathbf{v}_0$ abvZK X A‡¶i mv‡_ θ_0 †KvY Drcb K‡i (wPÎ : 3.7)|

myZivs X I Y A¶ eivei Gi Aw` †e‡Mi Dcvsk ¸‡jv n‡e

$\mathbf{V}_{x0} = V_0 \ \text{Cos}\theta_0$

$\mathbf{V}_{y0} = V_0 \ \text{Sin}\theta_0$ … … … … … … .. …. … … (3.14)



wPÎ t 3.8

†h †Kvb gyn~‡Z© t †Z e¯‘wUi †eM **v** ai‡j H †e‡Mi Dcvsk ¸‡jv v_x Ges v_y Avgiv mgxKiY (3.11) e¨envi K‡i cvB

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

ev, $v_x\,\mathbf{i} + v_y\,\mathbf{j} = v_{x0}\,\mathbf{i} + v_{y0}\,\mathbf{j} + a_x t\,\mathbf{i} + a_y t\,\mathbf{j}$

$= (v_0 \cos\theta_0)\,\mathbf{i} + (v_0 \sin\theta_0)\,\mathbf{j} + 0 - gt\,\mathbf{j}$

A_vr, $v_x = v_0 \cos\theta_0$ Ges $v_y = v_0 \sin\theta_0 - gt$ ... ... (3.15)

AZGe †`Lv hvq †h mgM DÇqbKvj e¨wc †e‡Mi x Dcvsk Z_v Abyf~wgK Dcvsk v_x AcwiewZZ _v‡K| †h †Kvb gyn~‡Z †eM **v** n‡j Gi gvb :

$$|\mathbf{v}| = v = \sqrt{v_x^2 + v_y^2} \quad \text{... ... ...} \quad (3.16)$$

Ges †eM **v** hw` Abyf~wg‡Ki mv‡_ A_vr X A‡¶i mv‡_ θ †KvY Drcb K‡i Zv n‡j

$$\tan\theta = \frac{v_y}{v_x} \quad \text{... ... ...} \quad (3.17)$$

Ges GB †e‡Mi w`K e¯‘wUi MwZ c‡\_i c‡Z¨K we›`y‡Z AswKZ ¯úkK eivei [†hgb wPÎ 3.8 G †`Lv‡bv n‡q‡Q] KvR K‡i|

†h †Kvb gyn~‡Z t ‡Z e¯‘wUi Ae¯vb †f±i **r** n‡j Gi x I y Dcvsk A_vr e¯‘wUi x I y ¯vbvsK Avgiv MwZi mgxKiY (3.13) †_‡K wbYq Ki‡Z cwi|

MwZ mgxKiY (3.13) n‡j,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0\, t + \frac{1}{2}\mathbf{a}t^2$$

ev, $$x\mathbf{i} + y\mathbf{j} = x_0\,\mathbf{i} + y_0\,\mathbf{j} + v_{x_0} t\mathbf{i} + v_{y_0} t\mathbf{j} + \frac{1}{2} a_x\, t^2\,\mathbf{i} + \frac{1}{2} a_y t^2\,\mathbf{j}$$

$$= 0 + 0 + (v_0 \cos\theta_0)\, t\mathbf{i} + (v_0 \sin\theta_0)\, t\mathbf{j} + 0 - \frac{1}{2}\, gt^2\,\mathbf{j}$$

ev, $x = (v_0 \cos\theta_0)t$ Ges $y = (v_0 \sin\theta_0)\, t - \frac{1}{2}\, gt^2$ ... ... (3.18)

myZivs †h †Kvb gyn~‡Z Ae¯vb †f±i **r** Gi gvb n‡jv

$$|\mathbf{r}| = r = \sqrt{x^2 + y^2} \quad \text{... ... ...} \quad (3.19)$$

Ges Ae¯vb †f±i **r** hw` Abyf~wgK ev x A‡¶i mv‡_ θ′ †KvY Drcb K‡i Zv n‡j,

$$\tan\theta' = \frac{y}{x} \quad \text{... ... ...} \quad (3.20)$$

cv‡mi MwZc‡_i mgxKiY t †Kvb e¯‘i MwZc‡_i mgxKiY n‡"Q †h †Kvb gyn~‡Z Zvi ¯vbvsK¸‡jvi m¤úK wb‡`kK mgxKiY| aiv hvK cv‡mi MwZc‡\_ GKwU we›`y p Gi ¯vbvsK (x,y) mgq t Gi A‡c¶K| mgxKiY (3.18) †_‡K t Gi A‡c¶K wn‡m‡e ¯vbvsK Gi gvb x I y cvIqv hvq|

GLb GB mgxKiY¸‡jv †_‡K t AcmviY Ki‡j x I y Gi g‡a¨ m¤úK cvIqv hv‡e| (3.18) mgxKiY `y‡Uvi c_gwU †_‡K cvß t Gi gvb wØZxqwU‡Z emv‡j wb‡Pi mgxKiYwU cvIqv hv‡e|

$$y = (v_o \sin\theta_o) \times \frac{x}{v_o \cos q_o} - \frac{1}{2} g \left(\frac{x}{v_o \cos\theta_o}\right)^2$$

ev, $$y = \frac{v_o \sin\theta_o}{v_o \cos\theta_o}\, x - \frac{1}{2}\, \frac{gx^2}{(v_0 \cos\theta_0)^2}$$

ev, $$y = x \tan\theta_o - \frac{1}{2}\left(\frac{g}{v_o^2 \cos^2\theta_o}\right) x^2 \quad \text{... ... ...} \quad (3.21)$$

GB mgxKiY †h †Kvb gvb‡Z x I y Gi m¤úK A_©vr Abyf~wgK I Dj¤^ ¯'vbvs‡Ki m¤úK wb‡`©k K‡i| GB mgxKiY n‡"Q cv‡mi Pj‡iLvi mgxKiY|

GB mgxKi‡Y

v_o, θ_o Ges g a"eK e‡j $\tan\theta_o$ Ges $\frac{g}{2v_o^2 \cos^2\theta_o}$ a"eK|

mZivs $\tan\theta_o = a$

Ges $\frac{g}{2v_o^2\cos^2\theta_o} = b$ ai‡j (3.21) mgxKiYwU `vovq

$$y = ax - bx^2 \quad \ldots \ldots \ldots \ldots \ldots (3.22)$$

hv GKwU Awae„‡Ëi mgxKiY| AZGe cv‡mi Pj‡iLv n‡"Q GKwU Awae„Ë|

m‡e©vaK D"PZv

m‡e©vaK D"PZvq e¯'i †e‡Mi উল্লম্ব Dcvsk k~b¨ nq, A_©vr $v_y = 0$ GB k‡Z mgxKiY (3.15) †_‡K t Gi †h gvb cvIqv hvq| aiv hvK t_m n‡e m‡ev"P D"PZvq IVvi mgq| A_©vr

$$v_y = v_o \sin\theta_o - gt$$

$$0 = v_o \sin\theta_o - g\, t_m$$

ev, $$t_m = \frac{v_o \sin\theta_o}{g} \quad \ldots \ldots \ldots \ldots \ldots (3.23)$$

†h †Kvb ¯'v‡b g a"e iwk e‡j $t_m \propto v_o \sin\theta_o$

mZivs †`Lv hvq †h m‡e©vaK D"PZvq IVvi mgq t_m e¯'i wb‡¶cb †e‡Mi উল্লম্ব Dcvs‡ki mgvbycvwZK|

$t = t_m$ mg‡q y Gi gvbB n‡"Q m‡e©vaK D"PZv h_m |

(3.18) mgxKi‡Y $y = h_m$ Ges $t = t_m = \frac{v_o \sin\theta_o}{g}$ emv‡q m‡e©vaK D"PZvi gvb cvIqv hvq,

$$h_m = v_o \sin\theta_o \times \frac{v_o \sin\theta_o}{g} - \frac{1}{2} g \left(\frac{v_o \sin\theta_o}{g}\right)^2$$

ev, $$h_m = \frac{v_o^2 \sin^2\theta_o}{g} - \frac{1}{2}\frac{v_o^2 \sin^2\theta_o}{g}$$

ev, $$h_m = \frac{(v_o \sin\theta_o)^2}{2g} \quad \ldots \ldots \ldots \ldots \ldots (3.24)$$

†h †Kvb ¯'v‡b g GKwU a"eiwk AZGe $h_m \propto (v_o \sin\theta_o)^2$ A_©vr GKwU cv‡mi m‡e©vaK †h D"PZvq D‡V Zv e¯'i Avw` †e‡Mi Dj¤^ Dcvs‡ki e‡Mi mgvbycvwZK| Avgiv Rvwb θ Gi gvb 90° Gi Rb¨ sinθ Gi gvb m‡e©vaK nq A_©vr sin90°= 1(m‡e©vaK)

AZGe $h_m = \frac{v_o^2}{2g}$ m‡e©vaK gvb ZLbB n‡e hLb e¯'‡K Lvov Dc‡ii w`‡K A_©vr 90° Dr‡¶cb †Kv‡Y wb‡¶c Kiv n‡e|

DÇqb Kvj

cv‡mi wb‡¶‡ci ci Averi fc,‡ô GKB Z‡j wd‡i Avm‡Z †h mgq jv‡M Zv‡K DÇqb Kvj ejv nq| e¯z fc,‡ô wd‡i Avm‡j উল্লম্ব miY kb¨ A_vr $y = 0$ nq| 3.18 mgxKi‡Y GB kZ emv‡j cvß t Gi gvb n‡e DÇqb Kvj|

g‡b Kwi DÇqb Kvj T

AZGe, $y = (v_o \sin\theta_o) \frac{1}{2} gt^2$ mgxKi‡Y Dj¤ miY, $y = 0$ Ges DÇqb Kvj $= T$ emv‡q

$$0 = (v_o \sin\theta_o) T - \frac{1}{2} gT^2$$

$$\therefore \quad T = 0 \text{ ev, } T = \frac{2v_o \sin\theta_o}{g} \quad \text{.. ... ...} \quad (3.25)$$

†h‡nZz $T = 0$ fc,ô †_‡K †h gyû‡Z e¯wU wb‡¶c Kiv n‡"Q †mB mgq wb‡`k K‡i| ZvB GwU Mnb †hvM¨ bq

myZivs $T = \frac{2v_o \sin\theta_o}{g}$ iwkwU e¯'i DÇqb Kvj wb‡`k K‡i|

†h‡nZz $\frac{2}{g}$ a"eK, myZivs $T \propto v_o \sin\theta_o$ A_vr DÇqb Kvj e¯'i wb‡¶cb †e‡Mi Dj¤ Dcvs‡ki mgvbycvwZK|

Avbyf,wgK পাল্লা

cm½ mgZ‡j wb‡¶cb I cZb we›`yi ga¨eZx `~iZ‡K cv‡mi পাল্লা e‡j| পাল্লা‡K R aiv nq| (3.21) mgxKi‡Y $y = 0$ emv‡j, পাল্লা $x=R$ cvIqv hv‡e|

(3.21) mgxKiYwU n‡"Q $y = (\tan\theta_0)x - \frac{g}{2(v_o \cos\theta_o)^2} x^2$

$y = 0$, $x=R$ emv‡q,

$$0 = (\tan\theta_o) R - \frac{g}{2(v_o \cos\theta_o)^2} R^2$$

$$\therefore R = 0 \text{ A\_ev } R = \tan\theta_o \times \frac{2(v_o \cos\theta_o)^2}{g}$$

†h‡nZz $R = 0$ †h ¯vb n‡Z e¯wU wb‡¶c Kiv nq †mB Ae¯vb wb‡`k K‡i ZvB GwU Mnb‡hvM¨ bq| AZGe,

$$R = \tan\theta_o \frac{2(v_o \cos\theta_o)^2}{g}$$

$$= \frac{\sin\theta_o}{\cos\theta_o} \cdot \frac{2v_o^2 \cos^2\theta_o}{g}$$

$$= \frac{2 \sin\theta_o \cos\theta_o}{g} \cdot v_o^2$$

$$R = \frac{v_o^2 \sin 2\theta_o}{g} \quad \text{. ... ...} \quad (3.26)$$

mewaK AvbyfywgK পাল্লা

Dc‡ii mgxKiY †_‡K †`Lv hvq g a"e nIqvq Aw`‡eM v_o *w¯i _vK‡j* R *Gi gvb wb‡¶cY †KvY* θ_o *Gi Dci wbfikxj|* R *mewaK n‡e hLb* $\sin 2\theta_o$ *mewaK n‡e|*

Avgiv Rvwb, $\sin 2\theta_o$ *mewaK* +1 *n‡Z cv‡i; †m‡¶‡Î* $\sin 2\theta_o = 1$ *ev,* $\theta = 45^0$

A_vr wbw`ó †e‡M wbw¶ß GKwU e¯zmewaK AbyfywgK `‚iZ AwZµg Ki‡e hLb e¯wU AbyfywgK‡Ki m‡½ 45^0 *†KvY wbw¶ß n‡e A_vr wb‡¶cb †KvY* 45^0 *n‡j* পাল্লা *mewaK n‡e|*

mewaK পাল্লার *gvb*

$$R_m = \frac{v_o^2 \sin 90^o}{g} = \frac{v_o^2}{g} \quad \text{... ... ...} \quad (3.27)$$

MwYwZK mgm¨ve jx

D`vniY 1

GKwU cv‡mi Aw` MwZ‡eM $20 ms^{-1}$ *Ges wb‡¶cb †KvY* 30^0 *cvmwUi (K) mewaK D"PZv KZ n‡e? (L)* পাল্লা *KZ? (M) wePiY Kvj KZ ?*

mgvavb t *†`qv Av‡Q* $v_o = 20\ ms^{-1}$; $\theta_o = 30^0$; $g = 9.8\ ms^{-1}$

(K) mewaK D"PZv h_m

$$h_m = \frac{(v_o \sin\theta_o)^2}{2g}$$

$$= \frac{(20 \times \sin 30^o)^2}{2 \times 9.8}$$

$$= \frac{(20 \times 0.5)^2}{2 \times 9.8}$$

$$= 5.1\ m$$

(L) AvbyfywgK পাল্লা R

$$R = \frac{v^2 \sin 2\theta_o}{g}$$

$$= \frac{(20ms^{-1})^2 \times (\sin 2 \times 30^o)}{9.8\ ms^{-2}}$$

$$= \frac{20^2\ m^2\ s^{-2} \times 0.866}{9.8\ ms^{-2}}$$

$$= 35.347\ m$$

(M) wePib Kvj $T = \dfrac{2v_o \sin\theta_o}{g}$

$$= \frac{2 \times 20\ ms^{-1} \sin 30^o}{9.8\ ms^{-2}}$$

$$= \frac{2 \times 20\ ms^{-1} \times 0.5}{9.8\ ms^{-2}}$$

$$= 2.04\ s$$

**D`vniY 2 t** *GKwU c‡mi Avbyf~wgK* পাল্লা 48m *Ges Aw` †eM* $33\ ms^{-1}$ *| wb‡¶cY †KvY KZ?*

mgvavb t *GLv‡b †`qv Av‡Q , Avbyf~wgK* পাল্লা $R = 48m$,

Aw`‡eM $v_0 = 33\ ms^{-1}$, $g = 9.8\ ms^{-2}$ *wb‡¶c †KvY* $\theta_0 = ?$;

Avgiv Rwb, $R = \dfrac{v_o^2 \sin 2\theta_o}{g}$

$$\therefore \sin 2\theta_0 = \frac{Rg}{v_o^2}$$

$$= \frac{48m \times 9.8ms^{-2}}{(33ms^{-1})^2}$$

$$= 0.43196$$

$$\therefore 2\theta_0 = \sin^{-1}(0.43196) = 25.6^o$$

$$\therefore \theta_0 = \left(\frac{25.6}{2}\right)^o = 12.8^o$$

mvims‡¶c

cvm t *†Kvb e¯‘‡K Avbyf~wg‡Ki mv‡_ wZhKfv‡e k~‡b¨ (evZv‡m) wb‡¶c Kiv n‡j Zv‡K cvm e‡j|*

cv‡mi MwZ wØgvwÎK| Gi MwZc_ Awae„ËvKvi| †h †Kvb wb‡¶cb †e‡Mi Rb¨ wb‡¶cb †KvY 45^0 *n‡j cv‡mi Avbyf~wgK* পাল্লা *mevaK nq| wb‡¶cb †KvY* 90^o *n‡j cvmwU* উল্লম্ব *fv‡e mevaK D"PZvq DV‡Z cv‡i|*

cv‡mi Pj‡iLvi mgxKiY nj $y = x \tan\theta_o - \dfrac{g}{2v_o^2 \cos^2\theta_o}\ x^2$

cv‡VvËi g~j¨vqb
mwVK DËiwUi cv‡k wUK wPý (√) w`b|

1| cv‡mi MwZ KZ gwÎK?
(K) GKgwÎK (L) wØ gwÎK
(M) wÎ gwÎK (N) gvÎvnxb

2| GKwU cv‡mi Zvr¶wYK †e‡Mi AwfgyL †Kvb w`‡K?
(K) উল্লম্ব w`‡K (L) AbyfzwgK w`‡K
(M) wePiY c‡_i j¤ eivei (N) wePiY c‡_i ¯úkK eivei

3| GKwU cv‡mi Pj‡iLv wK ai‡Yi?
(K) mij †iLv (L) e„Ë
(M) AwaeË (N) DceË

4| cv‡mi DÇqb Kvj m¤ú‡K †KvbwU mwVK?
(K) Aw`‡e‡Mi উল্লম্ব Dcvs‡ki mgvbycvwZK|
(L) Aw`‡e‡Mi AbyfzwgK Dcvs‡ki mgvbycvwZK|
(M) Zvr¶wYK †e‡Mi উল্লম্ব Dcvs‡ki mgvbycvwZK|
(N) Zvr¶wYK †e‡Mi AvbyfzwgK Dcvs‡ki e‡Mi mgvbycvwZK|

cvV - 3
e„Ëxq MwZ

D‡Ïk¨

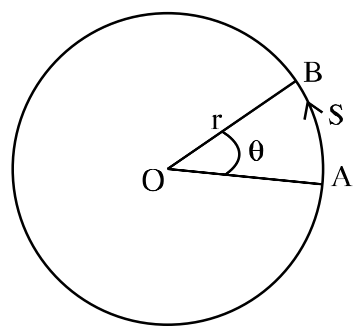
G cv‡Vi †k‡l Avcwb-

- e„Ëxq MwZi msÁv I D`vniY w`‡Z cvi‡eb,
- e„Ëxq MwZ‡Z ˆiwLK I †KŠwYK †e‡Mi aviYv cv‡eb Ges G‡`i g‡a¨ m¤úK© ¯’vcb Ki‡Z cvi‡eb,
- ˆiwLK I †KŠwYK †e‡Mi exRMwYZxK Ges †f±i i‚‡c m¤úK© cKvk Ki‡Z cvi‡eb|

3.3.1 e„Ëxq MwZ

†Kvb e¯‘ev KYv †Kvb wbw`©ó A¶‡K †K›`ª K‡i e„ËvKvi c‡_ AvewZZ n‡j Gi MwZ‡K e„ËvKvi MwZ ev e„Ëxq MwZ (Circular motion) e‡j|

e„Ëxq MwZ GK ai‡bi N~Y©b MwZ| N~Y©b hw` †Kvb A¶‡K †K›`ª K‡i m¤úbœ Zq Zv n‡j H A¶‡K N~Y©b A¶ (Axis of rotation) e‡j| N~Y©b hw` GKwU wbw`©ó †K›`ª K‡i m¯úbœ Zq Zv n‡j H wbw`©ó N~Y©b †K›`ª (Center of rotation) e‡j| GKwU KYv e„Ë c‡_ Nyi‡Z _vK‡j †K›`ª I KYvi Ae¯'vb ms‡hvMKvix mij‡iLv‡K e¨vmva© †f±i e‡j|



wPÎÑ3.9 (K)

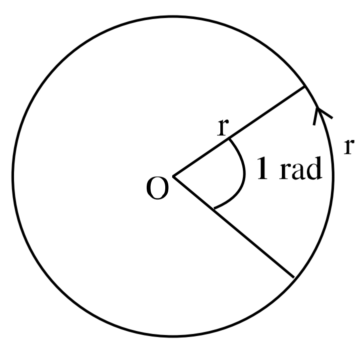
aiv hvK, GKwU e¯‘e„ËvKvi c‡_ Nyi‡Z Nyi‡Z †Kvb GK mgq A Ae¯'v‡b †_‡K B Ae¯'vb †cŠQvj [wPÎ 3.9 (K)]| e¯‘wUi GB Ae¯'v‡bi cwieZb‡K Avgiv `yfv‡e fve‡Z cwi|

1. e¯‘KYvwU e„‡Ëi cwiwai Dci A †_‡K B wbw`©ó AB = S `~iZ AwZµg K‡i‡Q| e„Ë Pvc S ‰iwLK `~iZ, hw` I GwU eµ †iLv| Gi cwigvc n‡e wgUvi GK‡K ev ˆ`‡N¨i GK‡K |

2. e¯‘wU e„‡Ëi †K‡›`ª †h †KvY Drcbœ K‡i Zvi mvnv‡h¨ Avgiv e¯‘wUi MwZ eYbv Ki‡Z cwi| G‡¶‡Î †K‡›`ª Drcbœ †KvY‡K †KŠwYK miY e‡j| e„Ëxq MwZi †¶‡Î †KŠwYK miY‡K θ Øviv cKvk Kiv hvq|

†KŠwYK miY θ = ∠AOB.

θ cwigv‡ci Rb¨ †iwWqvb GKK e¨envi Kiv nq| wWMÖx I e¨envi Kiv †h‡Z cv‡i| †Kvb e„‡Ëi e¨vmv‡ai mgvb e„Pvc †K‡›`ª †h †KvY Drcbœ K‡i Zv‡K GK †iwWqvb e‡j| wPÎ 3.9(L)



wPÎ 3.9 (L)

†Kvb e„‡Ëi cwiwa = 2πr A_©vr cwiwa‡K Avgiv e¨vmva© r Gi mgvb ˆ`‡Ni 2π msLK e„Pv‡c fvM Ki‡Z cwi|

GLb ˆ`‡N¨i e„Pvc e„‡Ëi †K‡›`ª †h †Kvb Drcbœ K‡i Zvi cwigvc 1 †iwWqvb

∴ GKK ˆ`‡N¨i (1) e„‡Pvc e„‡Ëi †K‡›`ª ‡KvY Drcb K‡i $\frac{1}{r}$ †iwWqvb

∴ $2\pi r$ e„‡Pvc e„‡Ëi †K‡›`ª †h †Kvb Drcb K‡i $\frac{1}{r} \times 2\pi r$ †iwWqvb

$= 2\pi$ †iwWqvb

AZGe, 1 AveZb = 2π †iwWqvb = 360 wWMÖx|

ms‡¶‡c $1\text{rev} = 2\pi \text{ rad} = 360^0$

†iwWqv‡b †KvY cwigv‡ci mgxKiY n‡"Q e„‡Ëi e¨vmva© Øviv †KvY‡K ¸b Ki‡j e„‡Pv‡ci ˆ`N¨ cvIqv hvq|

msÁvbymv‡i

r ˆ`‡N¨i e„‡Pvc Drcb K‡i 1 †iwWqvb †KvY

∴ 1 ˆ`‡N¨i e„‡Pvc Drcb K‡i $\frac{1}{r}$ ‡iwWqvb †KvY

∴ S ˆ`‡N¨i e„‡Pvc Drcb K‡i $\frac{S}{r}$ ‡iwWqvb †KvY

A_©vr $\frac{\text{Pvc}}{\text{e¨vmva}}$ = †Kv‡Yi e„Ëxq cwigvc

ev, θ (†iwWqv‡b) = $\frac{S}{r}$ ... ... 3.28(a)

ev, S (†iwWqv‡b) = θr ... ... 3.28(b)

cwiZ©b e„‡Pvc r ˆ`N©wewkó n‡q e„‡Ëi †K‡›`ª †iwWqvb †Kvb Drcb K‡i

∴ 2π e„‡Pvc r ˆ`N©wewkó n‡q e„‡Ëi †K‡›`ª †iwWqvb †Kvb Drcb K‡i 2π ‡iwWqvb

A_©vr m¤ú~Y© cwiwa †K‡›`ª 2π †iwWqvb †Kvb Drcb K‡i|

A_ev Gi cwiZ©b fvM e„‡Pvc †K‡›`ª 1 †iwWqvb †Kvb Drcb K‡i

2π fvM e„‡Pvc †K‡›`ª $1\times 2\pi$ †iwWqvb †Kvb Drcb K‡i

A_©vr m¤ú~Y© cwiwa eive‡i N~iv Avm‡j †K‡›`ª 2π †iwWqvb †Kvb Drcb n‡e|

GB mgxKi‡Yi mvnv‡h¨ Avgiv mn‡RB ˆiwLK I †KŠwYK `~i‡Zi g‡a¨ m¤úK© ¯'vcb Ki‡Z cvwi|

†KŠwYK †eM

†Kvb wb›`y ev A¶‡K †K›`ª K‡i e„ËvKv‡i c‡_ Pjgvb †Kvb e¯‘ †KŠwYK mi‡Yi nvi‡K †KŠwYK †eM e‡j| Ab¨fv‡e ejv hvq, N~Y©vqgvb e¯‘GKK mg‡q e„‡Ëi †K‡›`ª †h †KvY Drcb K‡i ZvB †KŠwYK †e‡Mi gvb ev †KŠwYK `ªæZ|

awi hvK, GKwU e¯‘ KYv r e¨vmv‡ai e„ËvKvi c‡_ Pj‡Z Pj‡Z Δt mgq AeKv‡k A Ae¯'vb †_‡K B Ae¯'v‡b Avmvq KYvwUi †KŠwYK miY nj $\angle AOB = \Delta\theta$ (wPÎ 3.10)

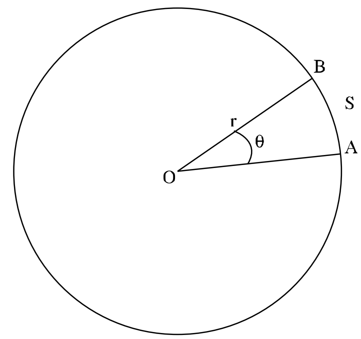
myZivs Mo †KŠwYK †eM, $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$ ... ... (3.29)

Δt k~b¨i mgxceZx n‡j $\bar{\omega}$ Gi mxgvšÍK gvb n‡e Zvr¶wYK †KŠwYK †eM

A_vr Zvr¶wYK †KŠwYK †eM

$$\omega = \lim_{\Delta t \to 0} \bar{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

ev, $\omega = \frac{d\theta}{dt}$ ... ... (3.30)



wPÎ 3.10

eE©vKvi c_wU m¤ú~Y© GKevi Nyi‡Z KYvwUi †h mgq jv‡M Zv‡K chvqKvj e‡j| †Kvb AveZbkxj e¯‘i chvq Kvj T n‡j,

$\omega = \frac{2\pi}{T}$ ... ... (3.31)

cwZ †m‡K‡Û e¯‘Zh¸‡jv c~Y© N~Y©b m¤úb K‡i Zv‡K K¤úvsK e‡j|

GK chvqKv‡j A_vr T mg‡q m¤úb nq 1wU N~Y©b

∴ GKK mg‡q m¤úb nq $\frac{1}{T}$ msL¨K N~Y©b

∴ $f = \frac{1}{T}$

A_vr $\omega = 2\pi f$ ... ... (3.32)

†KŠwYK †e‡Mi gvÎv : K¤úvsK‡K 2π w`‡q ¸Y Ki‡j †KŠwYK †eM cvIqv hvq| d‡j †KŠwYK †e‡Mi I K¤úvs‡Ki gvÎv GKB| GKvi‡Y †KŠwYK †eM‡K †KŠwYK K¤úvsKI ejv nq|

gvÎv : †KŠwYK †e‡Mi gvÎv n‡"Q, $\frac{\text{†KvY}}{\text{gvÎv}}$ Gi gvÎv|

†KŠwYK †eM = $\frac{\text{†KvY}}{\text{mgq}} = \frac{\text{Pvc}}{\text{e¨vmva} \times \text{mgq}}$ [∵ †KvY = $\frac{\text{Pvc}}{\text{e¨vmva}}$]

$\therefore [\omega] = \frac{L}{L \times T} = T^{-1}$

GKK : †KŠwYK †e‡Mi GKK = $\frac{\text{†KvY}}{\text{mgq}}$ Gi GKK = rad s^{-1}

‰iwLK `"wZ I †KŠwYK `"wZi m¤úK

aiv hvK r e¨vmv‡ai eE©vKvi c‡_ GKwU e¯‘ mg`"wZ‡Z Ny‡iQ| mgM eE©vKvi c_wU T †m‡K‡Û e¯‘wU GKevi Nyi Av‡m|

Zv n‡j e¯‘wUi AwZµvš— ‰iwLK `~iZ = cwiwa = 2πr Ges †KŠwYK `~iZ = 2πr

AZGe ˆiwLK `"wZ = $\frac{\text{cwiwa}}{\text{mgq}}$

ev, $v = \frac{2\pi r}{T}$ ... ... (3.33)

$$†KŠwYK\ `"wZ = \frac{†KvY}{mgq}$$

ev, $\omega = \frac{2\pi}{T}$ ... ... (3.34)

(3.33) mgxKi‡Y (3.34) mgxKiY cwZ¯vcb K‡i cvIqv hvq,

$$v = \frac{2\pi r}{T} = \frac{2\pi}{T} r$$

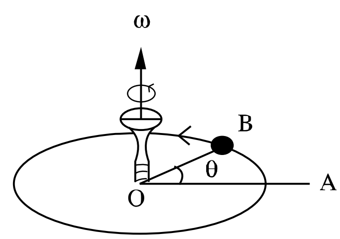
ev, $v = \omega r$ ... ... (3.35)

A_vr ˆiwLK `"wZ = †KŠwYK `"wZ × e¨vmva©|

†KŠwYK miY I †KŠwYK †e‡Mi †f±i iƒc

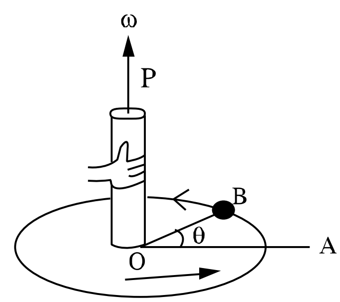
‰iwLK miY I †e‡Mi b¨vq †KŠwYK †eM I †KŠwYK miYI †f±i ivwk| myZivs †KŠwYK †e‡Mi Av‡jvPbvq w`‡Ki D‡ল্লখ Acwinvh©| eË©xq MwZ hw` GKwU we›`y‡K †K›`ª K‡i nq, Zvn‡j I GKwU N~Y©b A¶ Kíbv K‡i †bqv hvq| GB A¶, eË©c_ †h mgZ‡j Aew¯Z †mB mgZ‡ji j¤ eivei Ges †K›`ª Mvgx| eË©xq MwZmsµvš— †f±i mg‡ni Awfgyl GB A‡¶i w`‡K ev mgvšiv‡j aiv nq| GKwU D`vniY †`Iqv hvK|

GKwU Wvb nwZ ¯Œy mvnv‡h¨ †KŠwYK †e‡Mi w`K wb‡`k Kiv hvq| e„‡Ëi †K‡›`ª Awfj¤ fv‡e GKwU Wvb nwZ ¯Œz¯vcb K‡i eË©vKvi c‡_ e¯‘wU †h µ‡g Nyi‡Q †m µ‡g ¯Œz‡K Nyiv‡j ¯Œzh w`‡K G‡Mv‡e †mwUB n‡e †KŠwYK †e‡Mi w`K|



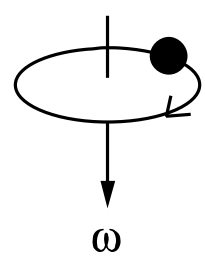
(wPÎ 3.11 K)

eB Gi mgZ‡j eË©vKvi c‡_ Pjvi mgq e¯‘wU hw` Nwoi KvUvi wecixZ w`‡K hvq Zvn‡j †KŠwYK †e‡Mi w`K n‡e eË©vKvi c‡_i †K‡›`ªi ga¨ w`‡q AvKv Awfj¤ eivei Dc‡ii w`‡K OP eivei (wPÎ 3.11 L)|



(wPÎ 3.11 L)

Nwoi KvUvi Nyi‡bi w`‡K e¯‘ N~Y©b n‡j †KŠwYK †e‡Mi Awfgyl n‡e wb‡Pi w`‡K (wPÎ 3.11M)|



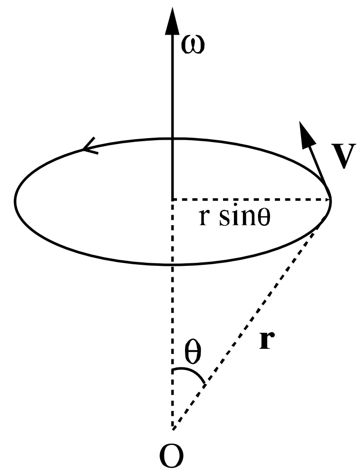
wPÎ 3.11 (M)

ˆiwLK †eM I †KŠwYK †e‡Mi m¤úK©

aiv hvK GKwU e¯‘KYv m¤ú~Y© KvVv‡gvi z A‡¶i Dci Aewe¯’Z O we›`yi †K›`ª K‡i XY mgZ‡j Nyi‡Q| e¯‘KYvwU e„ËvKvi c‡_ Nyi‡Q| †h †Kvb gyn~‡Z© e¯‘KYvi ˆiwLK †eM v e„‡Ëi ¯úk©K eivei|

Av‡iv aiv hvK e¯‘i †KŠwYK †eM ω| WvbnvZ¯’ wbqg †_‡K hvi w`K cvIqv hvq e„ËvKvi c‡_i Awf‡j¤^ Z_v z A¶ eivei| cm~Y© KvVv‡gvi g~j we›`y‡K †h †Kvb gyn~‡Z© e¯‘i Ae¯’vb †f±i r (wPÎ 3.12) n‡j e„ËvKvi c‡_i e¨vmva© = r sinθ

e¯‘i chvqKvj T n‡j Zvi ˆiwLK †e‡Mi gvb



wPÎ 3.12

$$v = \frac{2\pi\ r \sin\theta}{T} = \omega\ r \sin\theta \quad \ldots\ldots \quad (3.36)$$

wK¯‘ ω Ges r n‡”Q `yBwU †f±i Ges θ Zv‡`i AšÍf©y³ †KvY|

Kv‡RB `yBwU †f±i iwk¥i †f±i ¸Yd‡ji msÁv †_‡K cvB,

$$\omega r \sin\theta = |\boldsymbol{\omega} \times \mathbf{r}|$$

ev, $|\mathbf{v}| = |\boldsymbol{\omega} \times \mathbf{r}|$

Avevi `yBwU †f±i iwk¥i †f±i ¸‡Yi wbqgvbymv‡i ω×r Gi w`K Ges v Gi w`K GKB

$$\therefore\ \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \quad \ldots\ldots \quad (3.37)$$

†KŠwYK Z¦iY

†KŠwYK †e‡Mi cwieZ©b n‡j †KŠwYK Z¦iY nq| mg‡qi mv‡_ Amg †KŠwYK †e‡Mi cwieZ©‡bi nvi‡K †KŠwYK Z¦iY e‡j|

e„ËvKvi c‡_ Pjgvb e¯‘KYvi †KŠwYK †eM Δt mg‡qi AeKv‡k ω_i ‡_‡K ω_f G cwiewZ©Z n‡j Mo †KŠwYK Z¦iY,

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Ges Zvr¶wYK Z¦iY

$$\alpha = \underset{\Delta t \to 0}{\mathrm{Lt}} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad \ldots\ldots \quad (3.38)$$

†KŠwYK Z¦i‡Yi gvÎv $[\omega] = T^{-2}$

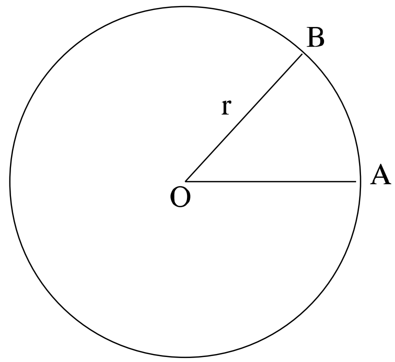
†KŠwYK Z¦i‡Yi GKK †iwWqvb/†m‡KÛ^2 (rad s^{-2})

‰iwLK ZiY I †KŠwYK Zi‡Yi g‡a¨ m¤úK

aiv hvK O *we›`yK †K›`ª K‡i GKwU e¯‘ e¨vmv‡ai eË¡vKvi c‡_ Nyi‡Q|* t_i *mg‡q e¯‘wU Ae¯’vb* A (*wPÎ 3.13*) *we›`y‡Z Ges Dnvi ˆiwLK †e‡Mi gvb* $\mathbf{v}_i$ *,†KŠwbK †e‡Mi gvb* $\boldsymbol{\omega}_i$

$\therefore\ v_i = \omega_i\ r$

aiv hvK t_f *mg‡q e¯‘wU* B *Ae¯’v‡b G‡m‡Q| GB Ae¯’v‡b ˆiwLK I †KŠwYK †eM h_vµ‡g* v_f *Ges* $\boldsymbol{\omega}_f$



wPÎ 3.13

$\therefore\ v_f = \boldsymbol{\omega}_f\ r$

GLb †e‡Mi cwieZ©‡bi Rb¨ ˆiwLK ZiY a *n‡j*

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{\omega_f r - \omega_i r}{t_f - t_i} = \frac{(\omega_f - \omega_i)r}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} r = \alpha r$$

$\therefore\ a = \alpha r$ ... . (3.39)

mvims‡¶c

eË¡xq MwZ t *†Kvb e¯‘ev KYv †Kvb we›`yev A¶‡K †K›`ª K‡i eË¡c‡_ AvewZZ n‡j Gi MwZ‡K eË¡xq MwZ ev eË¡vKvi MwZ e‡j|*

†KŠwYK †eM t *eË¡xq MwZ m¤úbœ †Kvb e¯‘i †KŠwYK mi‡Yi nvi‡K †KŠwYK †eM e‡j*

$$\boldsymbol{\omega} = \frac{d\theta}{dt}$$

†KŠwYK ZiY t *mg‡qi mv‡_ Amg †KŠwYK †e‡Mi cwieZ©‡bi nvi‡K †KŠwYK ZiY e‡j|*

$$\alpha = \frac{d\omega}{dt}$$

ˆiwLK †eM = †KŠwYK †eM × e¨vmva© †f±i

ev, $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$

MwwYwZK mgm¨vejx

**D`vniY 1t** GKwU †`qvj Nwoi wgwb‡Ui KvUvi ˆ`N¨ 18cm n‡j Gi †KŠwYK †eM Ges Gi cÖv‡šÍi ˆiwLK †eM wbYq Ki“b|

mgvavb t †`Iqv Av‡Q, chvqKvj T = 1hr = 3600 s

KvUvi ˆ`N¨ r = 18 cm = 0.18 m

†KŠwYK †eM ω = ?

‰iwLK †eM v = ?

Avgiv Rvwb, $\omega = \dfrac{2\pi}{T}$

$= \dfrac{2\times 3.14 \text{ rad}}{3600 \text{ s}}$

$= 1.74 \times 10^{-3}$ rad s^{-1}

Avevi v = ωr

$= 1.74 \times 10^{-3}$ rad s^{-1} $\times$ 0.18 m

$= 3.13 \times 10^{-4}$ ms^{-1}

**D`vniY 2 t** eµvKvi c‡_ 72 $kmh^{-1}$ mg `"wZ‡Z Pjgvb †Kvb Mvwoi †K›`ªgyLx ZiY 1 ms^{-2} n‡j eµvKvi c‡_i e¨vmva KZ?

mgvavb t †`qv Av‡Q,

`"wZ v = 72 kmh^{-1}

= 72×10^{3} m $\times$ $(3600\ S)^{-1}$

= 20 ms^{-1}

†K›`ªgyLx ZiY a = $1ms^{-2}$

e¨vmva r = ?

Avgiv Rvwb, $a = \dfrac{v^2}{r}$

$\therefore\ r = \dfrac{v^2}{a} = \dfrac{(20\ ms^{-1})^2}{1ms^{-2}} = 400$ m

**D`vniY 3 t** GKwU ˆe`y¨wZK cvLvi myBP ÔAbÕ Ki‡j 10 evi c~Y© N~Y©‡bi ci cvLwUi †KŠwYK †eM 20 $rads^{-1}$ nq †KŠwYK ZiY KZ? [a‡i wbb †h 10 evi c~Y© N~Y©‡bi I cvLwU mg †KŠwYK †eM cÖß nq]

mgvavb t †`Iqv Av‡Q 10 evi c~Y© N~Y©‡b †KŠwYK miY $\theta = 10 \times 2\pi$ rad = 20π rad, †kl †KŠwYK †eM ω_f = 20 rad s^{-1}; cÖv_wgK †KŠwYK †eM ω_i = 0 rad s^{-1}, †KŠwYK ZiY α = ?

Avgiv Rvwb, $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$2\alpha\theta = \omega_f^2 - \omega_i^2$

$$\therefore\ \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{(20 \text{ rad } s^{-1})^2 - 0}{2\times 20\pi \text{ rad}} = 3.183 \text{ rad } s^{-2}$$

cv‡VvËi g~j¨vqb

mwVK DË‡ii cv‡k wUK wPý (√) w`b|

1| †KŠwYK †e‡Mi gvÎv †KvbwU ?

(K) LT^{-1} (L) LT^{-2}

(M) T^{-1} (N) L^{-1}

2| †Kvb m¤úKwU mwVK bq?

(K) $\omega = 2\pi n$ (L) $\omega = \frac{2\pi}{T}$

(M) $v = \omega r$ (N) $v^2 = \omega r$

3| e„ËvKvi c‡_ Nyi‡j †KŠwYK †e‡Mi w`K †KvbwU wVK?

(K) †Kvb w`K bvB

(L) e„Ë mgZ‡j †K›` ei vei AvbyfwgK

(M) e‡Ëi †K›` Mvgx Ges Nyb Z‡ji Dci j¤ ei vei

(N) WvbnwZ ¯ŒziNy‡bi w`‡K|

4| GKwU KYv e„ËvKvi c‡_ Nyi‡Z _vK‡j †K›` I KYvi Ae¯vb ms‡hvMKvix mij‡iLv‡K wK e‡j?

(K) Nyb A¶ (L) e¨vmva

(M) e¨vmva †f±i (N) †f±i A¶

5| o †K›` Ges r e¨vmva wewkó e‡Ëi cwiwai Dci w`‡q MwZkxj e¯KYv Δt mgq e¨eav‡b A †_‡K B Ae¯v‡b Gj e¯KYvi ˆ`iwLK miY KZ?

(K) AB Pvc (L) $\angle AOB$

(M) $\frac{\Delta\theta}{\Delta t}$ (N) $\omega = \frac{\angle AOB}{\Delta t}$

cvV - 4
eË©xq MwZi mgxKiY I †K›`ªgyLx ZiY

D‡Ïk¨

G cv‡Vi †k‡l Avcwb

- *eË©xq MwZi ˆiwLK I †KŠwYK †e‡Mi cv_©K¨/Zzjbv Ki‡Z cvi‡eb,*
- *eË©xq MwZi mgxKiY¸wj cwZcv`b I eYbv Ki‡Z cvi‡eb,*
- *eË©xq MwZi †K›`ªgyLx ZiY e¨vL¨v I cwigvc Ki‡Z cvi‡eb|*

3.4.1 eË©xq MwZi ˆiwLK I †KŠwYK †e‡Mi Zzjbv

G BDwb‡Ui cȗeeZx© Ab¨‡”Q` ¸wji Av‡jvPbvq Avgiv †R‡bwQ eË©xq MwZ‡K ˆiwLK I †KŠwYK Dfq fv‡eB cKvk Kiv hvq| c~‡e©i c‡ôi Dci w`‡q A_ev mgZ‡ji Dci w`‡q GKwU RvnvR c‡_ †_‡K cwð‡g Pj‡Z Pj‡Z Av‡Mi RvqMvq wd‡i G‡j Zvi MwZ‡K Avgiv ˆiwLK fv‡eB cKvk Kwi| wKšzcKZ.c¯v‡e RvnvR‡Ri MwZ eË©xq MwZ, d‡j G‡K †KŠwYK MwZi iwki gva¨‡g I cKvk Ki‡Z cwi|

†KŠwYK I ˆiwLK †e‡Mi Zzjbv Ki‡Z †M‡j Avgiv Gi mv`„k¨ Ges cv_©K¨¸‡jv D‡ল্লখ Kie|

(K) mv`„k¨

1| DfqB †f±i iwk

2| AveZbiZ KYvi `~wZ evo‡j Df‡qiB gvb e„w× cvq

(L) cv_©K¨

ˆiwLK †eM	†KŠwYK †eM
1| ˆiwLK mi‡Yi nvi‡K ˆiwLK †eM e‡j|	*1| †KŠwYK mi‡Yi nvi‡K †KŠwYK †eM e‡j|*
2| ˆiwLK †e‡Mi gvÎv LT^{-1}	*2| †KŠwYK †e‡Mi gvÎv* T^{-1}
3| ˆiwLK †e‡Mi GKK ms^{-1}	*3| †KŠwYK †e‡Mi GKK* $rad\ s^{-1}$
4| mgZjxq GKgvwÎK I eË©xq MwZ Dfq †¶‡ÎB c‡hvR¨	*4| †Kej gvÎ eË©xq MwZi †¶‡Î c‡hvR¨|*
5| mg †KŠwYK †e‡M N~Y©biZ `„p e¯‘i wewfb KYvi ˆiwLK †eM wewfb|	*5| mg †KŠwYK †e‡M N~Y©biZ `„p e¯‘i c‡Z¨K KYvi †KŠwYK †eM mgvb|*

3.4.2 t eË©xq MwZi mgxKiY

Avgiv wØZxq Aa¨v‡q mij‡iLvq MwZkxj e¯‘i MwZi mgxKiYmgn~ cwZcv`b K‡iwQ| mgxKiY¸wj nj

i) $s = vt$ ii) $v = u + at$

iii) $s = ut + \frac{1}{2}at^2$ iv) $v^2 = u^2 + 2as$

GKB Dcv‡q Avgiv eË̈c‡_ MwZkxj e¯‘i †¶‡Î I Abyiƒc mgxKiY cÖZcv`b Ki‡Z cvie| Avmyb mgxKiY¸wj cÖZcv`b Kiv hvK|

(i) $\theta = \omega t$

msÁvbymv‡i t mg‡q †Kvb eË̈q c‡_ MwZkxj e¯‘ †KŠwYK miY θ n‡j, Mo †KŠwYK †eM $\omega = \frac{\theta}{t}$

ev, $\theta = \omega t$ . .. . (3.40)

(ii) $\omega_f = \omega_i + \omega t$

aiv hvK eË̈xq c‡_ MwZkxj e¯‘Ki †KŠwYK †eM t mgq AeKv‡k ω_i †_‡K †e‡o ω_f nj| Zvn‡j

msÁvbymv‡i, Mo †KŠwYK ZiY $\alpha = \frac{\omega_f - \omega_i}{t}$

ev, $\alpha t = \omega_f - \omega_i$

ev, $\omega_f = \omega_i + \alpha t$ ... . (3.41)

(iii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

g‡b Kwi eË̈xq c‡_ MwZkxj ‡Kvb e¯‘Ki cv_wgK †eM ω_i , t mgq e¨eav‡b †eM ω_f , †KŠwYK miY θ, Mo †KŠwYK †eM ω Ges mgZiY α

mgxKiY (3.40) †_‡K $\theta = \omega t$

$$\text{ev, } \theta = \frac{\omega_i + \omega_f}{2} \times t$$

$$\text{ev, } \theta = \frac{\omega_i + \omega_i + \omega t}{2} \times t$$

$$\text{ev, } \theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad \text{... .... (3.42)}$$

(iv) $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

(3.41) mgxKi‡Yi Dfq c¶‡K eM© K‡i cvB,

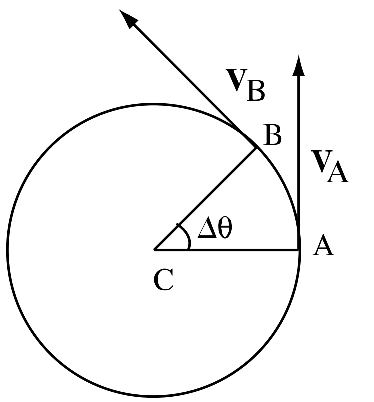
$\omega_f^2 = \omega_i^2 + 2\omega_i \alpha t + \alpha^2 t^2$

ev, $\omega_f^2 = \omega_i^2 + 2\alpha(\omega_i t + \frac{1}{2}\alpha t^2)$

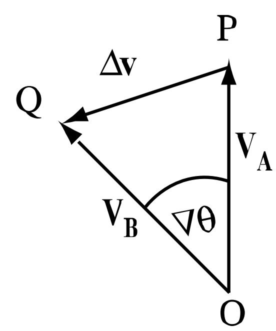
ev, $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ [3.41 n‡Z $\omega_f = \omega i + \alpha t$] .... (3.43)

3.4.3 mylg eËxq MwZ I †K›`ªgyLx ZiY

†Kvb e¯‘KYv hLb mg`ªæZ‡Z †Kvb eË©vKvi c‡_ Nyi‡Z _v‡K ZLb Zvi MwZ †e‡Mi gvb AcwieZ©Z _vK‡j I †e‡Mi AwfgyL cwiewZ©Z e`‡j hvq| MwZ‡eM GKwU †f±i ivwk, Gi gvb I w`K Av‡Q| gvb a"e _vK‡j ïaygvÎ w`K cwiewZ©Z n‡j I †e‡Mi cwieZ©b mwPZ nq| myZivs eŸ hv‡"Q †h, eË©vKvi c‡_ mg `ªæZ‡Z åvg¨gvb KYvi GKwU ZiY Av‡Q| Avgiv GLb GB Zi‡Yi gvb I w`K wbY©q Kie|



wPÎ 3.14 (K)



wPÎ 3.14 (L)

wPÎ : 3.14 (K) G mylg eË©vKvi MwZ‡Z MwZkxj GKwU e¯‘‡`Lvb n‡q‡Q| aiv hvK t mg‡q e¯‘KYwU e„‡Ëi cwiwai Dci A we›`y‡Z Ae¯'vb K‡i| G mg‡q Gi †eM $\mathbf{v}_A$ eË©wUi H we›`y‡Z AswKZ ¯úkK eivei wµqvkxj| Δt mg‡qi e¨eav‡b e¯‘wU B we›`y‡Z Gj| GB mgq Gi †eM $\mathbf{v}_B$ e„‡Ëi B we›`y‡Z AswKZ ¯úkK eivei wµqvkxj| aiv hvK A †_‡K B we›`y‡Z †h‡Z †K›`ª‡K mi Y Δθ Lyb ¶z|

e¯‘KYwU eËxq c‡_ mgvb `ªæZ‡Z Pj‡Q|

AZGe $\mathbf{v}_A$ Ges $\mathbf{v}_B$ Gi gvb mgvb| wKš‘ G‡`i AwfgyL Avjv`v|

wPÎ 3.14(L) †Z †Kvb we›`y O †_‡K $\mathbf{v}_A$ Ges $\mathbf{v}_B$ ‡f±i wb‡`kK †iLv h_vµ‡g OP Ges OQ †U‡b ΔOPQ m¤ú~Y© K‡i †f±i wÎfy‡Ri m~Î †_‡K ‡e‡Mi cwieZ©Y $\Delta\mathbf{v}$ wbY©q Kiv n‡q‡Q

$\mathbf{v}_B - \mathbf{v}_A = \Delta\mathbf{v}$ e¯‘KYvi `ªæZ mgvb e‡j, P I Q Lye KvQvKvwQ n‡j †jLv hvq,

OP = OQ = **v**

†h‡nZzΔθ †KvYwU Lye †QvU K‡RB Δ**v** Gi AwfgyL $\mathbf{v}_A$ Ges $\mathbf{v}_B$ Df‡qi m‡½B cvq j¤^| A_©vr A we›`y‡Z AC eivei Z_v e„‡Ëi †K›` C eivei e¯‘wUi †e‡Mi cwieZ©b ev ZiY nq| GB ZiY‡K †K›`ªgyLx ZiY e‡j|

%iwLK MwZ $\mathbf{v}_A$ Ges $\mathbf{v}_B$ Gi gvb mgvb A_©vr G‡`i gv‡bi †Kvb cwieZ©b n‡"Q bv| AZGe G‡¶‡Î A we›`y‡Z AC eivei ZiY, †e‡Mi w`‡Ki cwieZ©b NUvq| †K›`ªgyLx ejØqiv mµ GB ZiY e¯‘‡K eË©c‡_ Nyiqgvb †i‡L‡Q| GB ZiY bv _vK‡j e¯‘ ¯úkK eivei mij †iLvq mg‡e‡M Pj‡Z _vKZ|

wPÎ 3.14L †Z Δθ †KvYwU Lye ¶z ZvB $|\mathbf{PQ}| = |\Delta\mathbf{v}| = v\sin\Delta\theta = v\Delta\theta$ [wUKv -1 `óe¨]

GLv‡b v n‡"Q $\mathbf{v}_A$ Ges $\mathbf{v}_B$ Gi gvb, Av‡MB D‡ল্লখ Kiv n‡q‡Q e¯‘wU mg`ªæZ‡Z Ny‡Q e‡j Df q gvbB mgvb|

GLb †K›`ªgyLx ZiY $\mathbf{a}_r$ n‡j

$$\mathbf{a}_r = \underset{\Delta t \to 0}{\mathrm{Lt}} \frac{|\Delta\mathbf{v}|}{\Delta t}$$

$$= \underset{\Delta t \to 0}{Lt} \frac{v(\Delta\theta)}{\Delta t}$$

$$= v \underset{\Delta t \to 0}{Lt} \frac{\Delta\theta}{\Delta t}$$

$$= v \frac{d\theta}{dt}$$

$= v\omega$ .... (3.44)

wKšZ $v = r\omega$ ev, $\omega = \frac{v}{r}$

$\therefore$ $\mathbf{a}_r = \omega^2 r = \frac{v^2}{r}$ .... (3.45)

UxKv 1 t wPÎ 3.15 j¶¨ Ki"b CA Ges CB e‡Ëi `yÕwU e¨vmva©|

$\angle AOB = \theta$| awi hvK, θ AwZ ¶z`ª | $BO \perp CA$

†iwWqvb GK‡K $\theta = \frac{\text{Pvc AB}}{\text{e¨vmva©(r)}}$

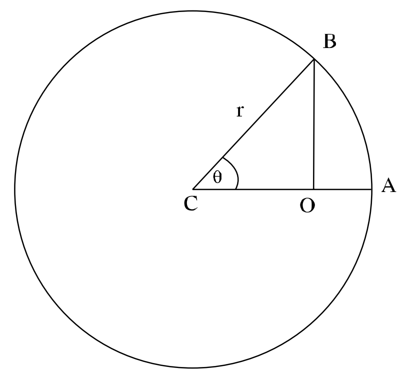
Avevi $\sin\theta = \frac{BO}{BC}$

θ AwZ ¶z`ª ZvB $BO \approx AB$ awi hvq|

$\therefore \sin\theta = \frac{\text{Pvc AB}}{\text{e¨vmva©(r)}}$

$\therefore \theta = \sin\theta$, hLb θ LyeB ¶z`ª

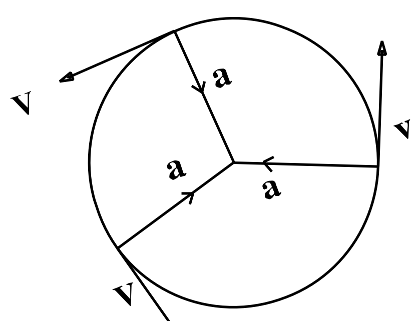
Avevi $\cos\theta = \frac{OC}{BC} \cong \frac{C}{BC} = \frac{r}{r} = 1$



wPÎÑ3.15

†K›`ªgyLx Z¡i‡Yi D`vniY

GKwU myZvi GK cÖv‡šÍ GKwU wXj †eu‡a Aci cÖvšÍ nv‡Z †i‡L wXjwU‡K e„ËvKvi c‡_ Nyiv‡bv hvq| G‡¶‡Î nvZ c‡qvRbxq ej cÖ‡qvM K‡i| GB e‡ji Rb¨ ‡K›`ªgyLx Z¡iY m„ó nq| e„Ëxq c‡_ NyY©qgvb e¯‘i ˆiwLK †e‡Mi gvb a"e _vK‡jI KYvwUi Dci G Z¡iY wµqv K‡i|



wPÎÑ3.16

KYvwU hLb Ny‡i Z _v‡K ZLb Gi ˆiwLK †e‡Mi w`K µgvMZ cwieZ©b nq| e„ËvKvi c‡_i †Kvb we›`y‡Z ˆiwLK †e‡Mi w`K H we›`y‡Z AswKZ ¯úk©K eivei| †eM †f±i ivwk| ZvB gvb a"e _vKv m‡Ë¡I w`K cwieZ©‡bi Rb¨ †e‡Mi cwieZ©b nq| †e‡Mi GB cwieZ©‡bi Rb¨ Z¡iY m„ó nq| e„ËvKvi c‡_ KYvi †h †Kvb Ae¯'v‡b G Z¡iY †K›`ªgyLx| [wPÎ 3.16]|

mvivsk

**†K›`ªgyLx ZiYt** e¯‘Ki MwZ‡Z MwZkxj c‡Z¨KwU e¯‘ ˆiwLK `¨wZ a‡e _vK‡j I Gi †K›`ªgyLx ZiY we`¨gvb| GB Zi‡Yi Drm †K›`ªgyLx ej hv e¯‘ MwZi w`‡Ki cwieZb NUvq| d‡j e¯‘ MwZ‡Z ˆiwLK ‡eM cwZ gyn~‡Z© w`K cwieZb K‡i A_vr MwZi gvb a‡e _vK‡j I ‡K›`ªgyLx Zi‡Yi d‡j MwZi w`K cwieZb nq| e¯‘ MwZi †¶‡Î MwZi mgxKiY¸wj nj :

(i) $\theta = \omega t$ (ii) $\omega_f = \omega_i + \alpha t$

(iii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ (iv) $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

Pevš— gyj¨vqb ckœgvjv

K. ˆbe©³K ck œ

mwVK DË‡ii cv‡k wUK wPý wPý (√) w`b

1| ZiY m¤ú‡K †KvbwU mwVK?

(K) $\mathbf{a} = \frac{d\mathbf{r}}{dt}$ (L) $\mathbf{a} = \frac{d\mathbf{s}}{dt}$

(M) $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ (N) $\mathbf{a} = \frac{d\boldsymbol{\omega}}{dt}$

2| cv‡mi †e‡Mi Abyf~wgK Dcvs‡ki Rb¨ †KvbwU mwVK

(K) GKwU wbw`ó cv‡mi Rb¨ gvb a“eK (L) mKj cv‡mi Rb¨ gvb a“eK

(M) gvb me`v cwieZbkxj (N) gvb g Gi Dci wbf©ikxj

3| †KŠwYK Zi‡Yi gvÎv †KvbwU ?

(K) T^{-1} (L) T^{-2}

(M) LT^{-2} (N) LT^{-1}

4| ˆiwLK †eM‡K e¨v‡Ëi e¨vmva© Øviv fvM Ki‡j wK cvIqv hvq?

(K) †KŠwYK †eM (L) †KŠwYK `¨wZ

(M) †KŠwYK ZiY (N) †KŠwYK fi‡eM

L. msw¶ß-DËi ck

1| msÁv wjL t cvm, cv‡mi পাল্লা, e¨vmva© †f±i, †KŠwYK miY, †KŠwYK †eM, chvqKvj, †KŠwYK ZiY, †K›`ªgyLx ZiY|

2| Mo †eM I Zvr¶wYK †e‡Mi †f±i i|c D‡ল্লখ Ki“b|

3| Zi‡Yi Dcvsk¸wj †_‡K Zi‡Yi gvb I w`K wKfv‡e Rvbv hvq ?

4| Zvr¶wYK Zi‡Yi w`K †Kvb w`‡K ?

5| cv‡mi Pj‡iLv wKi|c?

6| GKwU cv‡mi mewaK D”PZvq DV‡Z KZ mgq jv‡M?
7| cv‡mi mewaK D”PZvi mgxKiYwU wK?
8| cv‡mi DÇqb Kv‡ji mgxKiY †KvbwU ?
9| KZ †Kv‡Y †Kvb e¯‘‡K wb‡¶c Ki‡j Avbyf‚wgK পাল্লা mewaK nq?
10| †KŠwYK †e‡Mi GKK I gvÎv mgxKiY wjL|
11| †K›`ªgyLx Z¡i‡Yi gvb †Kvb †Kvb iwki Dci wbfi K‡i?
12| ˆiwLK ZiY I †KŠwYK Z¡i‡Yi g‡a¨ m¤úK© I m¤úK wK?
13| ˆiwLK †eM I †KŠwYK †e‡Mi Zzjbv Ki|
14| mewaK Avbyf‚wgK পাল্লার gvb KZ?

M. wek`-DËi cÖkœ

1| MwZi wb‡gv³ mgxKiY¸wj cÖZcv`b Ki|

(K) $v = v_0 + at$ (L) $r = r_0 + \frac{1}{2}(v_0 + v)t$ (M) $r = r_0 + v_0 t + \frac{1}{2}at^2$

2| GKgvwÎK MwZi mgxKiY¸‡jv e¨envi K‡i MwZi mgxKiY¸‡jvi †f±i iƒc cÖZcv`b Ki|
3| GKwU cv‡mi MwZc‡_i mgxKiY cÖZcv`b Ki|
4| †`Lvb †h GKwU cv‡mi Pj‡iLv n‡”Q AwaeÄ|
5| GKwU cv‡mi mewaK D”PZvq DVvi mgq, mewaK D”PZv I DÇqb Kv‡ji Rb¨ ivwkgvjv cÖZcv`b Ki|
6| $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ m¤úKwU cÖZcv`b Ki|
7| †K›`ªgyLx Z¡i‡Yi gvb I w`‡Ki Rb¨ ivwkgvjv wbYq Ki|
8| mg †KŠwYK Z¡i‡Y MwZkxj e¯‘i †¶‡Î cgvb Ki †h,

(i) $\omega_f = \omega_i + \alpha t$ (ii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ (iii) $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

N. MvwYwZK mgm¨v

1| GKwU cv‡mi Avbyf‚wgK পাল্লা 79.53 m Ges wePiYKvj 5.35 †m. wb‡¶cb †eM I wb‡¶cb †KvY wbYq Ki|
2| GKwU e¯‘‡K $4 ms^{-1}$ †e‡M Ges 35^0 wb‡¶cb †Kv‡Y k‚‡b¨ wb‡¶c Kiv n‡jv | KLb e¯‘wU i †e‡Mi AwfgyL Abyf‚wgK n‡e?
3| GKwU KYv 1.5 m e¨vmv‡ai GKwU eËvKvi c‡_ cwZ wgwb‡U 120 evi AveZb K‡i| KYvwUi (K) chvqKvj KZ? (L) †KŠwYK †eM KZ? (M) †iŠwLK †eM KZ?
4| nvZ Nwoi †m‡K‡Ûi KvUvi ˆ`N¨ 1.7 cm n‡j Gi cv‡šÍi ˆiwLK †eM wbYq Ki|
5| 0.05 Kg f‡ii GKwU e¯‘‡K 0.3 m `xN m~Zvi GK cv‡šÍ †e‡a eËvKvi c‡_ cwZ †m‡K‡Û 3 evi Nyiv‡bv n‡”Q| Gi †K›`ªgyLx ZiY KZ?