

BASIC FOUNDATION



Every subject/ course whether it relates to the disciplines of physical science, social science, arts, commerce (Business Administration), medical science, etc. has some basic foundations on which the course/subject is based and developed. Such basic foundations play the pivotal role in the growth and development of the course/subject. The present course entitled “**Fundamentals of Financial Management**” is also no exception to it. Therefore, the following basic foundations of the course are discussed in this **UNIT# ONE** :

1. Financial Management : Introductory Notes and Words (Lesson : 1)
2. Financial Decision Making Process (Lesson : 2)
3. Valuation Concepts : Time Value of Money (Lesson : 3)
4. Annuity : Time Value of Money (Lesson : 4)
5. Valuation of Long – Term Securities (Lesson : 5)
6. Measurement of Returns From Long – Term Securities (Lesson : 6)
7. Market Risk and Returns (MRR) (Lesson : 7)
8. Capital Asset Pricing Model (CAPM) and Expected Return and Risk (ERR) (Lesson : 8)

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Lesson–1: Financial Management: Introductory Notes and Words

After successfully completing this lesson 1, you should be able:

- To have a clear idea about the concept of financial management;
- To explain the relationship of financial management with other major areas of total management;
- To realize the significance of financial management in the context of industrial enterprises both public and private sectors, government and non-government organizations, educational institutions and the like;
- To identify the major goals and objectives of financial management and
- To understand agency relationship and control.

Concepts of Finance and Financial Management

Financial Management refers to the proper management of finance functions of an enterprise or organization. In other words, financial management is concerned with the financial decision-making and other financial aspects. Thus, financial management involves financial planning, financial organization, financial coordination and control, financial reporting, financial mergers, combinations and acquisitions, insurance and tax management etc. Financial planning is concerned with the act of deciding in advance the financial activities that are essential if the enterprises are to achieve their financial goals and objectives. These financial activities mainly consist of properly estimating financial needs; selecting the proper sources of finances; procuring the requisite funds; proper utilization of the funds and custody and safekeeping of funds. Financial organization is the grouping of the finance functions into various divisions, departments, sections and sub-sections of the enterprises for their proper and efficient performance. That is, financial organization deals with the proper allocation of the finance functions amongst the various financial executives. Financial coordination and control deal with the proper adjustment of the finance function and evaluation of the same in relation to the predetermined standards. Financial reporting is the proper collection and recording of financial data, preparing financial reports and statements and disseminating the same to the management for decision-making and other interest groups for information. Financial merger is a process whereby one enterprise is completely absorbed by another which may be achieved either by combination or acquisition. Insurance management deals with management of various insurance policies; while tax management deals with the management of tax aspects of an enterprise.

Financial management is concerned with the financial decision-making and other financial aspects.

Relation of Financial Management with Major Areas of Management

Financial management is the most significant aspect of total management of an enterprise. Management has its various aspects viz. marketing management, production management, material management/purchase

management, total quality management, human resource management and financial management. Of all these aspects financial management is the core one, since every other aspect either directly or indirectly involves finance. Therefore, financial management has relation with the other aspects of management.

Financial Management and Marketing Management

Financial management has a close relation with marketing management

Marketing is the total movement of goods and services from the original producer to the ultimate consumer. In such movement, many agencies and channels are involved. Marketing management deals with formulation of sales policy and target; selection of sales area; product types and types of customers; pricing of products and services; product promotions and advertisements; undertaking marketing research; selecting proper channels of distribution of products and services etc. In most of the marketing functions finance is involved directly or indirectly. As for example, promotion and advertisement activities require outlays of cash. Moreover, in conducting marketing research and in the channels of distribution the involvement of finance is a must. In these ways, it is seen that financial management has a close relation with marketing management.

Financial Management and Production Management

Financial management is also closely related with the production management of a manufacturing enterprise

Production management is mainly concerned with product development; formulation of production policies and targets; selecting proper production systems; following proper production control process; handling inventory problems; ensuring proper product quality etc. In most of the production functions of manufacturing enterprises, the involvement of finance is unavoidable. As for example, for purchasing raw materials a huge cash outlays is required. Moreover, for carrying out production cash outlays are needed for meeting various factory overheads as well as for direct or indirect labor. Thus, it is seen that financial management is also closely related with the production management of a manufacturing enterprise.

Financial Management and Material/ Purchase Management

Financial management is also closely related with materials/ purchase management

In case of manufacturing concerns material management is the vital one; while in case of trading concerns, the purchase management is very much significant. The purchase, movement, use and storage of production materials are fundamental management concerns in any manufacturing organization. Production material management deals with planning and scheduling of production functions; controlling the various types of inventories; purchasing materials and supplies; management of stores etc. In most of the material management functions, involvement of cash outlays is unavoidable. Production managers always try to make the best use of finance through careful scheduling of the sequence and timing of works and through the efficient control of buying, using and storing materials. Thus, it can be said that financial management is also closely related with materials/ purchase management.

Financial Management and Human Resource Management

People are the most valuable assets of an enterprise, which are not subject to depreciation like physical assets. Therefore, human resource management is the core of total management of an enterprise. Human Resource Management concerns with selection of the proper employees; methods of proper orientation and training of employees; policies and methods of remunerating the personnel; policies and procedures for the promotion and welfare of the personnel; policies and procedures for the transfer, hiring and firing of the employees; providing life and health insurance and pension policies of employees; system and criteria for evaluating employees' performances etc. In most of the personnel management areas, the involvement of finance is unavoidable. Hence, the question of proper management of finance functions also arises in the area of human resource management. Thus, it is seen that financial management is also closely related with human resource management.

Financial management is also closely related with human resource management

Significance of Finance and Financial Management

Financial management is the hard core of total management since every decision in an enterprise is ultimately a financial decision. The complex nature of the business enterprises demands that management is expected to give greater emphasis upon financial management of any enterprise whether trading, manufacturing, service rendering, government or non-government organizations, educational and charitable institutions and the like. Therefore, the following sub-sections examine the significance of financial management in the context of these organizations.

Industrial Organizations

In case of industrial organizations whether public or private sectors and whether small, medium and large; financial management is significant for the following main reasons :

- (i) Determining financial goals and objectives of both the short-term and long-term;
- (ii) Formulating financial policies both short-term and long-term;
- (iii) Financing both short-term and long-term;
- (iv) Determining proper capital and financial structure;
- (v) Determining cost capital of each of the sources;
- (vi) Formulating capital investment policy;
- (vii) Selecting proper methods and techniques of capital budgeting;
- (viii) Formulating working capital policy including cash, receivables and inventory policies;
- (ix) Formulating profit planning and control policies including dividend policies;
- (x) Developing proper financial information system and
- (xi) Formulating policies relating to maintenance of funds, insurance and tax and mergers, combinations and acquisitions.

Financial Management is Significant in cases of industries

Government and Non-government Organizations

Financial management plays a pivotal role in case of the government and her organizations and agencies for ensuring financial discipline

Financial management plays a pivotal role in case of the government and her organizations and agencies for ensuring financial discipline. For efficient and effective government financial administration, financial management has a great role to play. Therefore, financial management is of utmost importance for government; since government deals with public money and hence subject to public accountability. For government and in case of government agencies and organizations financial management is significant for the following important purposes:

- (i) Preparation of government budgets and getting the budgets passed by the competent authority/ legislation;
- (ii) Implementation of the budgets;
- (iii) Raising the requisite funds for the implementation of the development projects;
- (iv) Financing capital budgets of the government;
- (v) Formulating policies relevant to fund management, tax and insurance management;
- (vi) Selection of the major heads of revenue income of the government;
- (vii) Selection of the major heads of expenditures of the government;
- (viii) Treasury management i.e. safe custody of funds collected and due arrangement for the necessary payments to meet the liabilities and
- (ix) Arranging for proper accounting and audit of the government funds and other affairs.

Financial management is of utmost significance in cases of government educational institutions

Government Educational Institutions

An efficient and effective financial management is important not only in business and government organizations but also in educational institutions like schools, colleges and universities. A huge financial involvement is essential to run the public schools, colleges and universities. These institutions are run purely on public money and are also subject to public accountability; hence proper financial management practice is a must in these cases. Therefore, financial management is of utmost significance in cases of government educational institutions for the following main purposes :

- (i) Estimating the financial requirements during a particular financial year for smooth running of the institutions;
- (ii) Preparation of both the revenue and development budgets of the institutions and getting them approved by the concerned authority;
- (iii) Selecting the revenue sources of finance, both internal and external;
- (iv) Identifying the major heads of revenue expenditures;
- (v) Financing of development expenditures and
- (vi) Adjustment of advances, if any.

Major Goals and Objectives of Financial Management

Maximization of profits is the basic goal

Financial management of any business organization, whether large, medium and small has a basic goal of maximization of profits of the organizations; since financial management acts as the representative or

agent of their owners. The following sub-section examines the maximization of profits as the basic goal of financial management :

Maximization of Profits

Profit maximization generally refers to the increase in net profits of an enterprise during a particular year as compared to the previous year. But, increase in absolute figures of net profits is not significant if sales volume increases proportionately to the increase in net profits. Therefore, it is not the absolute profit maximization but the maximization of profitability is the basic goal of financial management. Profitability refers to net profits in terms of sales, investment, equity capital etc. But, profitability maximization goal has four main shortcomings viz.; (i) it is ambiguous having no precise connotation; (ii) it does not consider time value of money; (iii) it ignores the degree of certainty of benefits, that is, it ignores risk factor involved in earning profits and (iv) it does not always equate to more money in the shareholders' pockets and thereby fails to an increase in shareholders' stock price and, in turn, to shareholders' wealth. Therefore, profitability maximization may not be the proper goal of financial management; since the criterion is inappropriate and unsuitable as an operational objective of investment, financing and dividend decisions of a firm. In order to remove the shortcomings involved in the profitability maximization goal; wealth maximization goal has been evolved and also recognized by the authors of financial management as well as by the professionals. Such goal is examined in sub-section 1.4.2.

Profit maximization generally refers to the increase in net profits.

Wealth Maximization

This criterion is also known as value maximization or net present worth maximization. Wealth/ value maximization is almost universally accepted as an appropriate operational decision criterion for financial management decisions for the following main grounds :

- (i) It has a definite connotation in the sense that cash flow is considered here as a measure of benefit which has a precise concept;
- (ii) It takes into consideration both the quantity and quality dimensions of benefits. In such criterion, necessary adjustments are made in the cash flow pattern, firstly to incorporate risk and secondly, to make an allowance for differences in timing of benefits;
- (iii) It incorporates the time value of money. The value of a stream of cash flow is calculated by discounting its element back to the present at a capitalization rate that reflects both time and risk.
- (iv) Since wealth is measured by the share price; it leads to an increase in the share price and hence in the shareholders' value /wealth if the financial management takes those actions or decisions that will increase share price. Therefore, the skill of financial managers must lie in their ability to identify those investments and financing that would increase the share price.

Wealth/ value maximization is almost universally accepted as an appropriate operational decision criterion for financial management decisions

Social Goals

Financial management has some social goals to achieve in order to increase the welfare of its employees, in one hand, and increase the image of the organization, on the other.

Besides the above discussed goal, financial management has some social goals to achieve in order to increase the welfare of its employees, in one hand, and increase the image of the organization, on the other. Such social goals assume the social responsibility of the firm. Social responsibility of the firm comprises the following :

- i) Protecting consumers' interest;
- ii) Paying fair wages and salaries and other fringe benefits to the employees;
- iii) Maintaining fair staff hiring practices and safe and congenial working conditions;
- iv) Providing requisite training to the existing staff and supporting education to their children;
- v) Involving in environmental issues like fresh air and pure water;
- vi) The interests of the stakeholders consisting of creditors, suppliers, customers, financiers, communities and the relevant government agencies should also be given due care and weightage.

Agency Relationship and Control

The shareholders' goal is to maximize their wealth; whereas the management's goal is to maximize their own welfare. So, there is conflict between these two goals and such conflict of goal is known as agency problem or agency cost.

The relationship that exists between organization, its owners or shareholders and management is known as agency relationship. The management team is the agent that is, they are hired on behalf of the shareholders. The goals of these parties are different. The shareholders' goal is to maximize their wealth; whereas the management's goal is to maximize their own welfare. So, there is conflict between these two goals and such conflict of goal is known as agency problem or agency cost. In such a context, the conflict of goals must be removed or minimized. There are two ways for removing/ minimizing the negative impact of the agency problem. The first is to ensure that the management objectives do not conflict with shareholders' objectives. The second is to use the ultimate control of the firm by exercising voting rights of the board of directors to remove a manager who is not acting in the best interest of the shareholders.

Review Questions

A. Short Questions

1. Give the concept of the term Financial Management.
2. Why financial management is said to be the core of total Management?
3. “The objective of a company must be to create value for its shareholders” – Explain the statement logically.
4. “Maximization of wealth goal is superior to maximization of profit goal” – Explain.
5. Why social goals of financial management are significant?
6. What is agency problem? How it can be solved?

B. Broad Questions

1. Examine the relation of financial management with the following major areas of management:
(i) Marketing management (ii) Production management
(iii) Material management (iv) Human Resource Management
2. Examine the significance of Financial Management in the context of the following organizations:
(i) Industrial organizations (ii) Government organizations
(iii) Educational institutions
3. Explain the major goals and objectives of financial management.

Lesson–2: Financial Decision Making Process

After carefully reading this Lesson 2, you should be able

- To discuss the fundamentals of financial decision making, explaining the financial decision making process;
- To realize the proper role of top level financial executives of the enterprises and
- To have an overview of the major financial decision making process.

Fundamental Areas of Financial Decision Making

The performance of these functions requires the professional knowledge and skill of the executives

The finance functions involving decision-making are known as financial decision-making functions. The performance of these functions requires the professional knowledge and skill of the executives. Such decision making finance functions are broadly categorized into three groups namely : (i) investment decision; (ii) financing decision and (iii) dividend decision. The following sub-sections deal with these decisions briefly.

Investment Decision

.Investment decision is the proper allocation of capital, both fixed and working to the investment projects whose benefits are to be realized in the future

The investment decision is the most significant of these three decisions when it is considered as the creation of value. Investment decision is the proper allocation of capital, both fixed and working to the investment projects whose benefits are to be realized in the future. Investment decision broadly includes the following main aspects:

Category – A : Long-term Investment Decision

i) Capital Budgeting Decision – It is a multi-dimensional activity which embraces searching for new and more profitable investment proposals; investigating engineering and technical, financial, economic, marketing and management considerations to predict the consequences of accepting on investment proposal in order to examine whether the investment proposal is viable. Any capital budgeting decision is a major financial decision and it is significant for a company for the three basic reasons such as : (a) it entails a huge amount of cash outlays; (b) it involves risk and uncertainty and (c) it affects the company's operation for a larger period of time.

ii) Analysis of risk and uncertainty

iii) Analysis of cost of the specific sources of fixed capital

Category – B : Short-term Investment Decision

i) Short-term financial objectives,

ii) Working capital investment policy namely cash, inventory marketable securities and receivables policies and

iii) Working capital control.

Financing Decision

After investment decision, the question of financing decision arises. Such decision consists of the following two main aspects :

i) Capital structure decision, which involves determining the best mix of equity, preferred stock, long-term debt and hybrid securities to employ. In such decision cost of each source of capital must be considered carefully. Moreover, the factors that affect the determination of capital structure must be given due weightage.

Besides determining capital and financial structures, financing decision includes the selection of the sources of funds and raising of funds from these sources

ii) Financial structure decision which is nothing but a financing mix consisting of shareholders' equity, preferred stock, long-term and short-term debts and hybrid securities. In case of such decision the cost of capital must be given due weightage.

While determining capital and financial structure of a firm its financial management should see whether the firm is under-capitalized or over-capitalized. Both the under-capitalization and over-capitalization have harmful effects on the form. Therefore, determination of an optimal capital and financial structure is a must for a firm if it would like to continue over a longer period of time.

Dividend Decision

The final important financial decision of an enterprise is dividend decision. It mainly involves : (i) formulation of profit plan, (ii) formulation of dividend policy, (iii) formulation of retention policy and (iv) investment of accumulated profits.

It mainly involves : (i) formulation of profit plan, (ii) formulation of dividend policy, (iii) formulation of retention policy and (iv) investment of accumulated profits.

While taking dividend decision of a firm, its financial management must give due considerations on the following aspects :

- a) Current earnings;
- b) Preference of the shareholders as to the current dividend income or future capital gains;
- c) Amount that should pay as dividend, that is, dividend payout ratio;
- d) Constraints on paying dividends;
- e) Retention ratio, that is, amount to be retained;
- f) Stable dividend policy – to follow or not;
- g) Forms of dividend – cash or bonus share and
- h) Stock split vs. stock dividend.

Financial Decision Making Process

The process with which the financial management of a firm will take the above discussed financial decisions is known as financial decision making process. The process starts from the initial establishment of the firm and ends with the closure of the firm. That is, the process continues during the lifecycle of the firm. At the time of initial establishment i.e. at the gestation period; the firms have to take investment decision. During the operating period, the firms have to take new investment decision, if needed; financing decision and dividend decision.

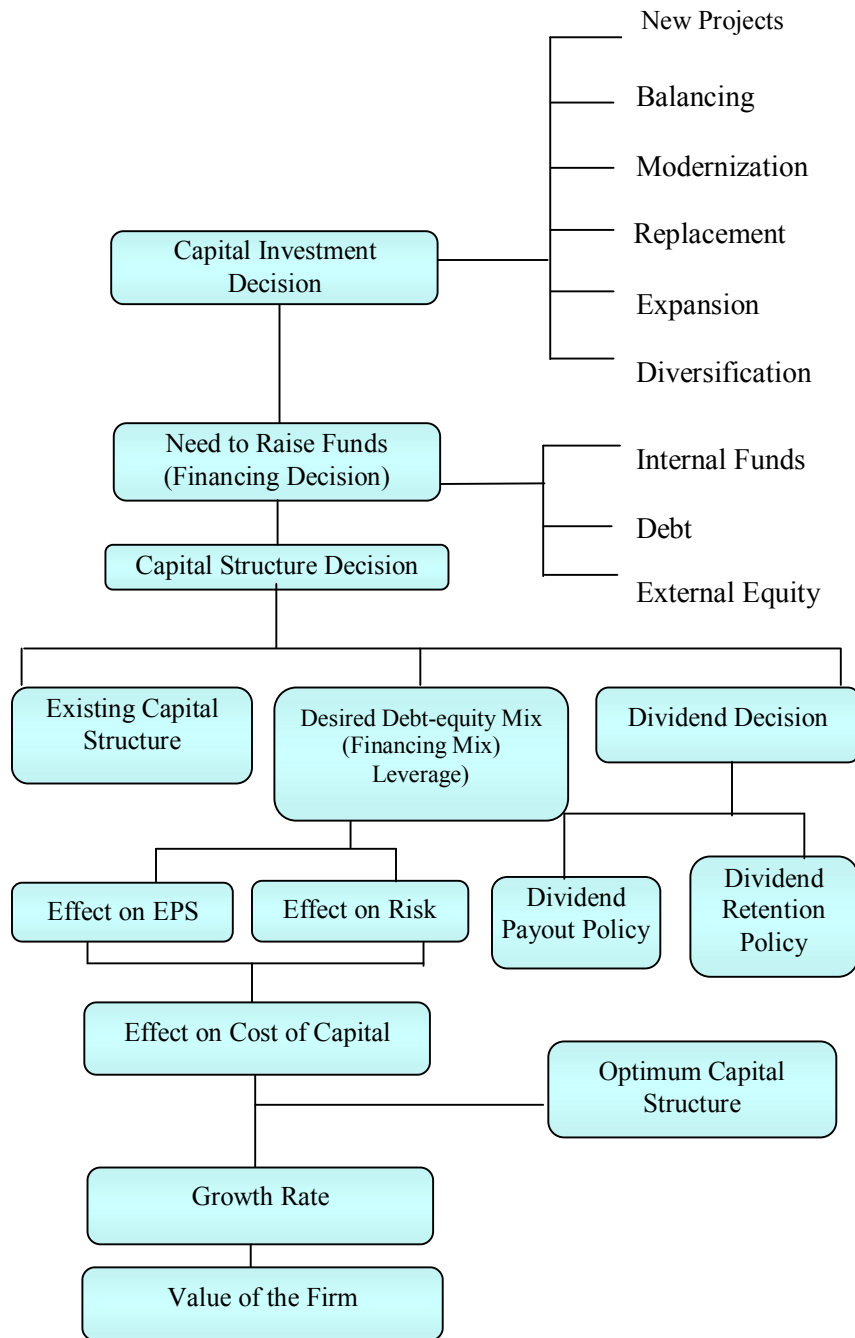
The process with which the financial management of a firm will take the above discussed financial decisions is known as financial decision making process

The proposition of the theory of a firm's finance is the capital structure theorem which specifies the relationship between the firm's capital structure and its cost of capital. From the theorem follow other propositions concerning the relationship between the firm's investment decision, financing decision and dividend decision, its cost capital and

market value. The process is shown in Chart-1, explaining the relationship between firm's financial decisions.

Chart – 1

Relationship between Firm's Major Financial Decisions



Role of Financial Executives in Decision Making Process

Financial executives starting from finance director down to cashier, a junior financial executive have a great role to play in the corporate firms. Their roles are discussed in the following sub-sections:

Status of the Top level Financial Executives

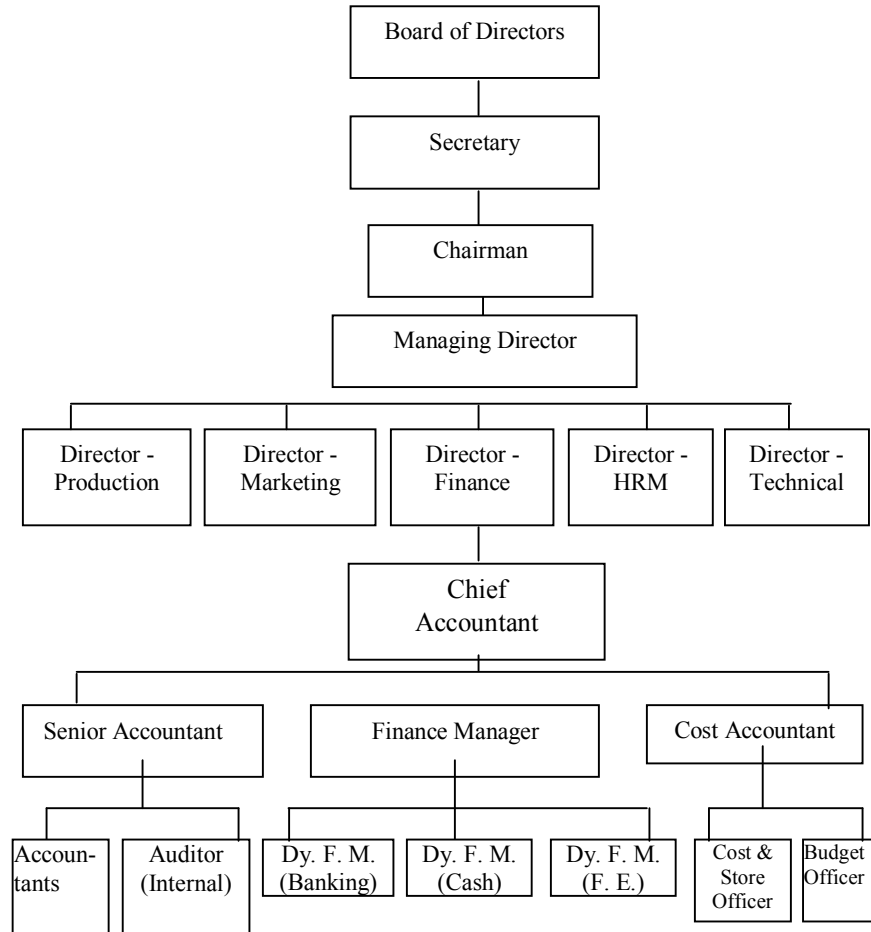
Financial executives are those personnel of the organization who are involved in the finance functions of that organization. That is, financial executives are responsible for the performance of financial functions. The financial executives who are involved in the executive or decision making finance functions are known as top level financial executives. In the performance of these executive finance functions, the financial executives have to possess executive skill i.e. specialized knowledge and skill. But, the financial executives who are involved in the incidental or non-decision making finance functions are known as mid and / or low level financial executives. In the performance of these finance functions the executives need not have to possess specialized knowledge and skill.

Top level financial executives are the key and responsible personnel of the organizations since they are involved in the decision makings. All the financial decisions are taken by the top level financial executives. Their status is exhibited in Chart-2 in case of a large private industrial enterprise in Bangladesh.

Top level financial executives are the key and responsible personnel of the organizations since they are involved in the decision makings

Chart - 2

Status of Top Level Financial Executives in Financial Organization Structure



Source: Organization Manual of the Selected Private Sector Industrial Enterprises in Bangladesh.

In Chart-1, it is evident that the topmost financial executive in the private sector industrial enterprises is the Director – Finance. Although he ranks equal status of other Directors viz., Production, Marketing, HRM and Technical; he holds central position among the directors in the organizational hierarchy. This is because of the fact the Director – Finance is the head of finance functions of the enterprise and in all the other directorates finance is involved, directly or indirectly. Under the Finance Director, there is the Chief Accountant under whose direct supervision and control there are departmental heads namely Senior Accountant for General Accounts, Finance Manager for Finance and Cost Accountant for Cost,, Stores and Budget Departments. Although they possess the same status like other departmental heads under the Production, Marketing, HRM and Technical Directorates; they hold

comfortable positions in the organizational hierarchy, since they are entrusted with the financial decision making functions, the key functions of the enterprises. Under the direct supervision and control of the departmental heads, there are sections namely Accounts (General), Audit (Internal), Cash, Banking, Foreign Exchange, Cost, Store and Budget. Although these section chiefs have the same status like other section chiefs under Production, Marketing, HRM and Technical Departments; their positions in the organizational hierarchy are higher since they deal with the finance.

Functions and Responsibilities of the Top Level Financial Executives

The finance functions of an enterprise can be grouped into two namely executive finance function or treasury finance function and incidental finance function or controllership finance function. All the executive finance functions are decision making finance functions and some of the controllership functions are also decision making functions. These functions have to be performed by the top level financial executives.

All the executive finance functions are decision making finance functions and some of the controllership functions are also decision making functions

The important executive finance functions are mentioned below :

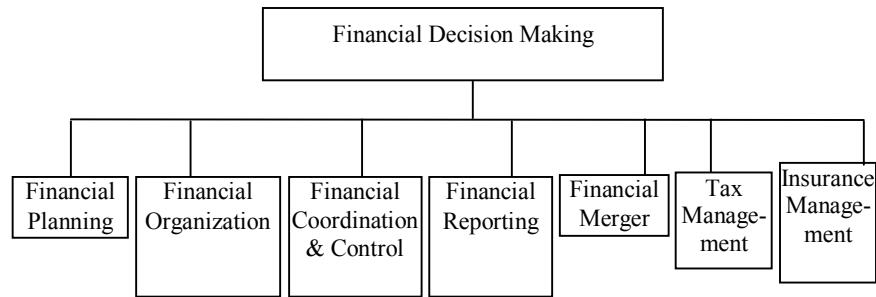
- i) Determination financial goals and objectives;
- ii) Formulation of financial policies;
- iii) Forecasting of cash flows;
- iv) Determination of fixed assets management policies;
- v) Determination of current assets management policies;
- vi) Determination of capital and financial structure;
- vii) Selection of sources of capital;
- viii) Raising of funds;
- ix) Credit management;
- x) Cash management;
- xi) Inventory management;
- xii) Receivable management;
- xiii) Marketable Securities Management
- xiv) Control of cost of capital;
- xv) Control of working capital;
- xvi) Control of inventory;
- xvii) Profit planning and control;
- xviii) Cost control;
- xix) Determination of dividend policy;
- xx) Insurance and tax management;
- xxi) Financial merger and acquisition and
- xxii) Product/ service pricing.

Overview of Financial Decision Making Process

While explaining the concept of financial decision making, the major processes of it has been identified. However, this process of financial management is shown in Chart-3.

Chart - 3

Major Process of Financial Decision Making



The following sub-sections deal with the major steps involved in each of the financial decision making process:

Financial Planning

Financial planning involves the following three fundamental steps :

- i) Determining both long-term and short-term financial goals and objectives;
- ii) Formulating as well as promulgating both long-term and short-term financial policies;
- iii) Developing strategies and procedures that aid in the promulgation of the financial policies;

Financial Organization

This financial organization process involves the following main steps :

- i) Grouping of financial functions into divisions, departments and sections etc.;
- ii) Allocation of financial functions amongst financial executives;
- iii) Entrusting financial powers and duties amongst the financial executives and
- iv) Fixing financial accountability and responsibilities of the financial executives

Financial Coordination and Control

This process of financial management involves the following main steps:

- i) Coordination of financial functions between the financial executives;
- ii) Determination of operational standards relating to financial functions;
- iii) Evaluation of the enterprises' actual performances in relation to pre-determined standards;
- iv) Instigation of corrective actions in case of deviations and
- v) Follow-up actions to ascertain if corrective actions are effective

Financial Reporting

The following major steps are involved in this process of financial management:

- i) Collection and recording of financial data;
- ii) Processing and analyzing of financial data;
- iii) Preparation and publication of financial reports and statements;
- iv) Circulation of financial reports and statements to interest groups.

Financial Merger

The following major steps are included in this process of financial management:

- i) Determining legal framework and also identifying legal process involved in mergers;
- ii) Determining the price considerations of the acquired or merged company;
- iii) Identifying the methods to be followed in mergers;
- iv) Determining the mode of payment of purchase consideration.

Tax Management

This process includes the following main steps of financial management:

- i) Following the appropriate tax policy by the government of the country;
- ii) Proper assessing of the enterprises in respect of income tax, VAT, etc. and
- iii) Evolving appropriate tax accounting system in the enterprises.

Insurance Management

The following major steps are involved in this process of financial management:

- i) Introducing and following appropriate insurance system and
- ii) Determining appropriate insurance policy to be taken by the enterprises.

Review Questions

A. Short Questions

1. Who are financial executives of an enterprise ? Classify them in terms of administrative hierarchy.
2. What are the executive finance functions ? Give examples.
3. What are the incidental finance functions ? Give examples.

B. Broad Questions

1. Examine the following financial decisions in the context of a large-scale industrial enterprise -
 - i) Investment decision
 - ii) Financing decision
 - iii) Dividend decision
2. Examine the relationship between firm's major financial decisions with the help of a chart.
3. Discuss the role of top-level financial executives of a large industrial organization.
4. Briefly discuss major steps involved in the following financial management process :
 - i) Financial Planning
 - ii) Financial Organization
 - iii) Financial Coordination and Control
 - iv) Financial Reporting
 - v) Financial Merger

Lesson-3: Valuation Concepts: Time Value of Money

After successfully completing lesson 3, you should be able:

- To have a clear concept on time value of money and other relevant values;
- To know the tools and techniques involved in determining present, future or terminal values and
- To solve for time and interest rates for present and future values.

Concept of Time Value of Money and other Relevant Values

In the literature of Finance and Mathematics, time value of money concept has been recognized. The concept signifies that money has time value. That is, the value of money varies in terms of time. According to this concept, a dollar received today is worth more than a dollar expected to be received in the future. This is because of the fact that the sooner a dollar is received, the quicker it can be invested to earn a positive return. Therefore, it is true that one dollar in the future is less valuable than one dollar of today. The relationship between one dollar in the future and one dollar of today is known as the time value of money. This present value concept of time value of money should be clearly understood by the investors as well as financial managers in order to examine its impact on the value of an asset.

The relationship between one dollar in the future and one dollar of today is known as the time value money

Future value or terminal value and present value are associated with present value of money. The following paragraphs deal with these values.

Future Value or Terminal Value

Future value is the value of a cash flow or a series of cash flows at some time of a present amount of money. That is, future value refers to the amount to which a cash flow or a series of cash flows will grow over a given period of time. Therefore, the future value is dependent on three things : i) present value; (ii) period and (iii) rate of interest. Thus, the future value at the end of one year equals the present value multiplied by one plus interest rate. As for example, if present value equals to Tk. 100, period is 1 year and rate of interest is 10 percent; then future value will be Tk. 110.

Future value refers to the amount to which a cash flow or a series of cash flows will grow over a given period of time

Present Value

Present value is the value today of a future cash flow or series of cash flows. That is, present value is a future amount discounted to the present by some required rate. The present value is dependent on three things : (i) future value, (ii) period and rate of interest. As for example, if future value is Tk.115, period is one year and rate of interest is 15 percent; then present value will be Tk. 100 only.

Present value is the value today of a future cash flow or series of cash flows

Since, cash flow is involved in both the future value and present value; it needs clarification. Cash flow embraces both cash outflow and cash

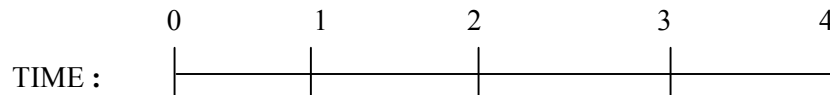
inflow. Cash outflow is a payment or disbursement of cash for expenses, investments and so on. On the other hand, cash inflow is a receipt of cash from an investment, an employer, a banker or from any other sources.

Tools and Techniques of Time Value of Money

Tools used in Time Value of Money

Cash outflow is a payment or disbursement of cash for expenses, investments and so on. On the other hand, cash inflow is a receipt of cash from an investment, an employer, a banker or from any other sources

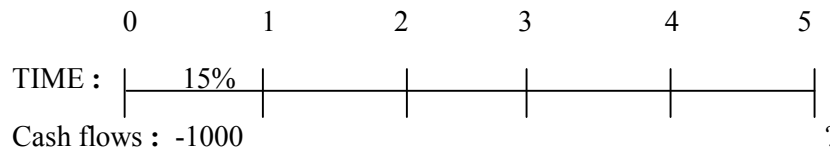
One of the most important tools in time value of money analysis is the cash flow time line. It is a graphical representation used to show the timing of cash flows. Such line is used helping us visualizing when the cash flows associated with a particular situation. Constructing a cash flow time line will help us to solve problems related to the time value of money. This is because of the fact that it illustrates what happens in a particular situation, making it easier to set up the problem for solution. To illustrate the time line concept, let us consider the following diagram.



The above diagram shows that time 0 is today, time-1 one period from today or the end of the period-1; time-2 is two periods from today or the end of period-2 and so on. Thus the values on the top of the tick marks represent end of period values. Often the periods are years, but other time intervals like semi-annuals, quarters, months or even days are also used.

Cash flows are placed directly below the tick marks and interest rates are shown directly above the cash flows time line. Unknown cash flows which need to be found out in the analysis are indicated by question mark. As for example, consider the following time line.

It is a graphical representation used to show the timing of cash flows



In the above diagram, the interest rates for each of the five periods is 15%; a single amount or lump sum cash flows are made at time-0; and the time-5 value is an unknown inflow. Because, the initial Tk. 1000 is a cash outflow or an investment, so it has a minus sign. But, the period-5 amount is a cash inflow; so it does not have a minus sign. Note that no cash flows occur at times-1, 2, 3 and 4. Also note that we do not show Taka signs on time lines; this reduces clutter.

The cash flow time line is an essential tool for better understanding time value of money concepts. The financial experts use cash flow time line to analyze the complex problems.

Techniques of Time Value of Money

The following two techniques are generally used in time value money : (i) compounding and (ii) discounting. The following paragraphs deal with each of the techniques.

Compounding Technique

A Taka in hand today is worth more than a Taka to be received in the future. This is because of the fact that if you had it now, you could invest it, earn interest and end up with more than one Taka. The process of going from today's values which are termed as present values (PV), to future values (FV) is called compounding. That is, the process of determining the value of a cash flow sometime in the future, by applying compound interest rate is known as compounding. By compound interest we mean interest earned on interest.

The process of determining the value of a cash flow sometime in the future, by applying compound interest rate is known as compounding. by compound interest we mean interest earned on interest

Compound Interest vs. Simple Interest

Compound interest refers to the interest earned on both the initial principal and the interest reinvested from prior periods, while simple interest refers to the interest earned only on the original principal amount invested. Let us clear these with examples. Suppose your principal amount is Tk. 1000 and the rate of interest is 10% and the period is 3 years. In the example, compound interest comes to Tk. 331 (100+110+121); whereas, simple interest comes to Tk. 300 (100+100+100) only at the end of 3 years.

Now, the question arises how the FVs are determined. There are two approaches to determine FVs: one is Equation approach and the other is Tabular approach.

In case of annual (single) compounding:

Under Equation Approach:

$$FV_n = PV (1+i)^n$$

Where, FV_n = Future value at period n

PV = Present value

i = Rate of interest

n = Time period

Under Tabular Approach:

$$FV_n = (1 + i)^n = PV(FV IF i, n)$$

Where, FV_n = Future value at period n

PV = Present value

IF = Interest factor to be found out from Future Value Table

i = Rate of interest

n = Time period

Terms of Interest and Future Values

Interest may be paid annually, semiannually, quarterly, monthly, even daily and even continuously or infinitely and such mode of payment of interest is known as terms of interest.

Interest may be paid annually, semiannually, quarterly, monthly, even daily and even continuously or infinitely and such mode of payment of interest is known as terms of interest. Interest may be paid annually, semiannually, quarterly, monthly, even daily and even continuously or infinitely and such mode of payment of interest is known as terms of interest. Such terms of interest have impact on the FVs. In the above Equation and Tabular Approaches of calculating FVs, we have assumed that interest is paid annually. Now, let us consider the relationship between FVs and interest rates for different periods of compounding. FVs and terms of interest have direct relationship, implying that the number of times interest paid in a year (m) is increased, the FV also increases. For different terms of interest, the formula for finding out FVs under both the Equation and Tabular Approaches need to be adjusted as follows :

a) In case of Multiple Compounding

Under Equation Approach

$$FV_n = P_v \left(1 + \frac{i}{m}\right)^{mn}$$

Under Tabular Approach

$$FV_n = \left(1 + \frac{i}{m}\right)^{mn} P_v (FVIF \frac{i}{m}, mn)$$

b) In case of continuous or infinite compounding

Under Tabular Approach

$$FV_n = P_v (e^{i \times n})$$

FVIF_{i,n} refers to the future value of Tk. 1 left on deposit for n periods at a rate of i percent per period that is, the multiplier by which an initial investment grows because of the interest earned.

Where, e is the value equal to 2.7183

Future Value Interest Factor for i and n (FVIF_{i,n})

FVIF_{i,n} refers to the future value of Tk. 1 left on deposit for n periods at a rate of i percent per period that is, the multiplier by which an initial investment grows because of the interest earned. In order to find out IF from Future Value Table, time period (n) and rate of interest (i) should be considered simultaneously. In the Table, the vertical column represent n; whereas, the horizontal columns represent rates of interest.

Problems and solutions

Problem – 1

Find out the future values (FV) in the following situations :

a) At the end of 3 years, how much is an initial deposit of Taka 1,000 worth, assuming a quarterly compounded interest rate of (i) 10% and (ii) 100%.

b) At the end of 10 years, how much is an initial investment of Taka 1,000 worth, assuming an interest rate of 10% compounded : (i) annually; (ii) semiannually; (iii) quarterly, (iv) monthly and (v) continuously ?

Solution :

a) In this problem, given PV = Tk. 1,000; n = 3years and i = 10% percent; 100%; required finding out FV.

Under Equation Approach

Under Tabular Approach

(i) $FV_n = PV(1+i)^n$

(i) $FV_n = PV (FVIF_{i,n})$

Where, FV_n = Future value at n period;

1,000(FVIF_{10%,3})

PV = Present value

1,000 x 1.3310

i = Interest rate and

Tk. 1,331.

n = Time period

= 1,000(1 + .10)³

(ii) $FV_n = PV(FVIF_{i,n})$

= 1,000(1.10)³

= 1,000(FVIF_{100%,3})

= 1,000 x 1.331

= 1,000 x 8.000

= Tk. 1,331

= Tk. 8,000.

(ii) Again, $1,000(1 + 1)^3$

= 1,000 x 8

= 8,000.

b) In this problem, given PV = Tk. 1,000; n = 10 years and i = 10%; what is FV.

Solution :

Under Equation Approach

Under Tabular Approach

<p>In case of annual interest :</p> <p>i) $FVn = PV(1+i)^n$ $= 1000(1+0.10)^{10}$ $= 1,000 \times 2.594$ $= \underline{\underline{Tk.2,594}}$</p>	<p>i) $FVn = PV(FVIF_{i,n})$ $= 1,000(FVIF_{10\%,10})$ $= 1,000 \times 2.594$ $= \underline{\underline{Tk.2,594}}$</p>
<p>In case of semiannual interest :</p> <p>ii) $FVn = PV\left(1 + \frac{i}{m}\right)^{mn}$ $= 1000\left(1 + \frac{10}{2}\right)^{2 \times 10}$ $= 1000(1 + 0.05)^{20}$ $= 1000 \times 2.6533$ $= \underline{\underline{Tk.2,653}}$</p>	<p>ii) $FVn = PV(FVIF_{\frac{i}{m}, mn})$ $= PV(FVIF_{5\%,20})$ $= PV(2.6533)$ $= \underline{\underline{Tk.2653}}$</p>

<p>In case of quarterly interest :</p> <p>iii) $FVn = PV\left(1 + \frac{.10}{4}\right)^{4 \times 10}$ $= 1000(1 + .025)^{40}$ $= 1000(2.6851)$ $= \underline{\underline{TK.2,685}}$</p>	<p>iii) $FVn = PV(FVIF_{\frac{i}{m}, mn})$ $= 1000(FVIF_{2.5\%,40})$ $= 1000 \times 2.6851$ $= \underline{\underline{TK.2,685}}$</p>
<p>In case of monthly interest :</p>	
<p>iv) $FVn = PV\left(1 + \frac{.10}{12}\right)^{12 \times 10}$ $= 1000(1 + .00833)^{120}$ $= 1000(2.7059)$ $= \underline{\underline{TK.2706}}$</p>	<p>iv) $FVn = PV(FVIF_{\frac{i}{m}, n})$ $= 1,000 (2.7059)$ $= \underline{\underline{TK.2,706}}$</p>

In case of Compounding Interest:

$$\begin{aligned}
 v) FVn &= PV (e^{ixn}) \\
 &= (2.7183)^{10 \times 10} \\
 &= \text{Tk. } \underline{2,718.30}
 \end{aligned}$$

[**Note** : In cases of FVIF value has not been provided in the Future Value Table. So, in these cases FVIF has been calculated by using the

alternative formula viz. $FVIF = \left(1 + \frac{i}{m}\right)^{mn}$]

Problem - 2

Assume that it is now January 1, 2000. On January 1, 2001, you will deposit Tk. 1000 into a Savings Account of Janata Bank that pays 12 percent interest per annum.

Required :

- (a) If the bank compounds interest annually how much will you have in your account on January-1, 2006 ?
- (b) What would your January-1, 2005 balance be if the bank used quarterly compound ?
- (c) Suppose you deposited Tk. 1000 in payments of Tk. 200 each on January 1, 2001, 2002, 2003, 2004 and 2005. How much would you have in account on January-1, 2005, based on 10 percent annual compounding ?

Solution :

Under Equation Approach

Under Tabular Approach

$$\begin{aligned}
 (a) FVn &= PV (1 + i)^n \\
 &= 1000 (1 + 0.12)^5 \\
 &= 1000 \times 1.7623 \\
 &= \text{Tk. } \underline{1762}
 \end{aligned}$$

$$\begin{aligned}
 FVn &= PV (FVIF_{i, n}) \\
 &= 1000 (FVIF_{12\%, 5}) \\
 &= 1000 \times 1.76723 \\
 &= \text{Tk. } \underline{1762}
 \end{aligned}$$

(b) Under Equation Approach

Under Tabular Approach

$$\begin{aligned}
 FVn &= PV \left(1 + \frac{i}{m}\right)^{mn} \\
 &= 1000 \left(1 + \frac{.12}{4}\right)^{4 \cdot 4} \\
 &= 1000 (1.03)^{16} \\
 &= 1000 \times 1.60477 \\
 &= \underline{\underline{\text{Tk. } 1605}}
 \end{aligned}$$

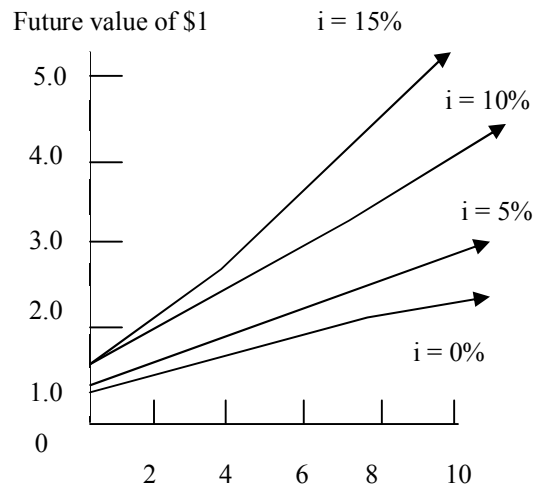
$$\begin{aligned}
 FVn &= PV (FVIF_{\frac{i}{m}, mn}) \\
 &= PV (FVIF_{3\%, 16}) \\
 &= 1000 \times 1.6047 \\
 &= \underline{\underline{\text{Tk. } 1605}}
 \end{aligned}$$

(c) You may solve this problem by finding the future value of an annuity of Tk. 200 for 5 years at 10 percent:

$$\begin{aligned} FV_n &= PMT (FVIFA_{i,n}) \\ &= \text{Tk. } 200(FVIFA_{10\%,5}) \\ &= \text{Tk. } 200 (6.1051) \\ &= \text{Tk. } \underline{1,221.} \end{aligned}$$

Graphic View of the Compounding Process : Growth

Figure -1 shows Tk. 1 or any other sum grows over time at various interest rates. The higher the rate of interest, the faster the rate of growth. The interest rate is, in fact, a growth rate. An example is shown in figure-1 which indicates relationships among future value, growth, interest rates and time.



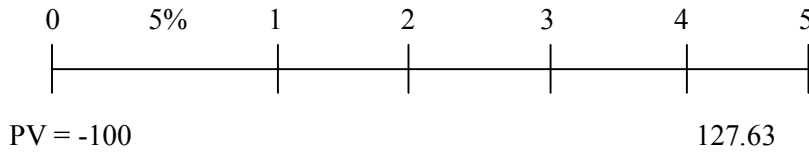
If a sum is deposited and earns 5 percent interest, then the funds on deposit will grow at a rate of 5 percent per period. Again, if a sum is deposited and earns 10 percent interest, then the funds on deposit will grow at a rate of 10 percent per period and so on. Also note that time value of money concepts can be applied to anything that is growing – sales, purchases, inventory, population, earning per share and so on.

Discounting refers to the process of determining the present value of a cash flow or a series of cash flows. It is the reverse of compounding.

Discounting Technique

Discounting refers to the process of determining the present value of a cash flow or a series of cash flows. It is the reverse of compounding. That is, the process of finding present values from future values is called discounting. If you know the FVs, you can discount the PVs. At the time of discounting you would follow these steps.

CASH FLOW TIME LINE



In the above figure it is seen that Tk. 100 would grow to Tk. 127.63 in 5 years at a 5 percent interest rates. Therefore, Tk. 100 is the PV of Tk. 127.63 due in 5 years in the future when the opportunity cost rate is 5 percent.

Determining Pvs Through Discounting

Like determination of FVs, PVs can also be determined by Equation Approach and Tabular Approach.

Under Equation Approach

Under Tabular Approach

i) $PV = \frac{FVn}{(1+i)^n}$

(In case single interest payment)

PV (Single payment) =

$$FVn \left[\frac{1}{(1+i)^n} \right] = FVn(PVIF_{i,n})$$

ii) $PV = \frac{FVn}{\left(1 + \frac{i}{m}\right)^{mn}}$

(In case multiple interest payment in a year like semiannually, quarterly, monthly and daily.)

PV (Multiple payment) =

$$FVn \left[\frac{1}{\left(1 + \frac{i}{m}\right)^{mn}} \right] = FVn(PVIF_{\frac{i}{m}, mn})$$

iii) $PV = \frac{FVn}{e^{i \times n}} = FVn(e - i \times n)$

(In case continuous or infinite compounding)

Where *e* is the value equal to 2.7183.

Present Value Interest Factor (PVIF)

Present value interest factor for *i* and *n* (PVIF_{*i*,*n*}) refers to the present value of Tk. 1 due *n* periods in the future discounted at *i* percent per period. In order to find out IF from Present Value Table, time period (*n*) and rate of interest (*i*) should be considered simultaneously. In the Table, the vertical column represents *n*; whereas, the horizontal column represent rates of interest.

*Present value interest factor for *i* and *n* (PVIF_{*i*,*n*}) refers to the present value of Tk. 1 due *n* periods in the future discounted at *i* percent per period.*

Problems and Solutions

Problem - 3

Determine the Present Values (PVs) in the following cases :

- a) Taka 1,000 at the end of 5 years is worth how much today, assuming a discount rate of : (i) 10 percent and (ii) 100 percent;
- b) What is the aggregate PVs of the following receipts, assuming a discount rate of 15 percent :
 - i) Taka 1,000 at the end of 1 year;
 - ii) Taka 1,500 at the end of 2 years;
 - iii) Taka 1,800 at the end of 3 years;
 - iv) Taka 2,200 at the end of 4 years and
 - v) Taka 2,500 at the end of 5 years ?

Solution

a) (i) In case of 10 percent discounting rate :

Under Equation Approach

Under Tabular Approach

$PV = \frac{FVn}{(1+i)^n}$ $= \frac{1000}{(1+.10)^5}$ $= \frac{1000}{1.6105}$ $= \underline{TK \ 620.93}$	$PV = FVn (PVIF_{i,n})$ $= 1,000 (PVIF_{10\%,5})$ $= 1,000(.6209)$ $= \underline{TK. 620.9}$
---	--

(ii) In case of 100 percent discounting rate :

Under Equation Approach

Under Tabular Approach

$PV = \frac{FVn}{(1+i)^n}$ $= \frac{1000}{(1+1)^5}$ $= \frac{1000}{325}$ $= \underline{TK \ 31.25}$	$PV = FVn (PVI F_{i,n})$ $= 1,000 (PVI F_{100\%,5})$ $= 1,000(.031259)$ $= \underline{TK. 31.25}$
---	---

b) Under Equation Approach Under Tabular Approach

<p>i) $PV = \frac{FVn}{(1+i)^n} = \frac{1000}{(1+.15)^1} = \underline{TK. 869.57}$</p>	<p>i) $PV = FVn (PVI F_{i,n})$ $= 1,000 (PVI F_{15\%,1})$ $= 1,000(.8696)$ $= \underline{TK. 869.60}$</p>
<p>ii) $PV = \frac{FVn}{(1+i)^2} = \frac{1500}{(1+.15)^2} = \frac{1500}{1.3225} = \underline{TK. 1134.22}$</p>	<p>ii) $PV = FVn (PVI F_{15\%,2})$ $= 1,500 (.7561)$ $= \underline{TK. 1134.15}$</p>
<p>iii) $PV = \frac{FVn}{(1+i)^3} = \frac{1800}{(1+.15)^3} = \frac{1800}{1.5209} = \underline{TK. 1183.51}$</p>	<p>iii) $PV = FVn (PVI F_{15\%,3})$ $= 1,800 (.6575)$ $= \underline{TK. 1183.50}$</p>
<p>iv) $PV = \frac{FVn}{(1+i)^4} = \frac{2200}{(1+.15)^4} = \frac{2200}{1.7490} = \underline{TK. 1257.86}$</p>	<p>iv) $PV = FVn (PVI F_{15\%,4})$ $= 2,200 (.5718)$ $= \underline{TK. 1257.96}$</p>
<p>v) $PV = \frac{FVn}{(1+i)^5} = \frac{2500}{(1+.15)^5} = \frac{2500}{2.0114} = \underline{TK. 1242.92}$</p>	<p>v) $PV = FVn (PVI F_{15\%,5})$ $= 2,500 (.4972)$ $= \underline{TK. 1243.00}$</p>

Hence, aggregate PVs = Tk. 869.57 + Tk. 1,134.22 Hence, aggregate PVs= Tk. 869.60

+ 1,183.51 + Tk. 1,257.86 + 1,242.92 = Tk. 5,688.08 + 1,134.15 + 1,183.50 + 1,257.96 +

Tk. 1,243.00 = Tk. 5,688.21

Problem - 4

Find the present values of the following amount due :

- Taka 6,600 due in 10 years at a 6 percent discount rate, calculating annually;
- Taka 9,000 due in 8 years at a 12 percent discount rate, calculated semiannually;
- Taka 12,000 due in 6 years at a 18 percent discount rate, calculated quarterly and
- Taka 15,000 due in 3 years at a 12 percent discount rate, calculated monthly.
- Taka 18,000 due in 5 years at a 15 percent discount rate, calculated continuously.

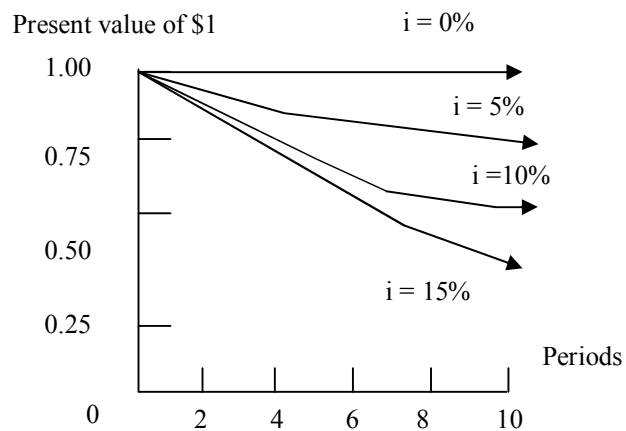
<p>a) $PV = \frac{FVn}{(1+i)^n} = \frac{6,600}{(1+.06)^{10}} = \frac{6,600}{1.7908} = \underline{TK. 3,685.50}$</p>	<p>a) $PV = FVn (PVI F_{i,n})$ $= 6,600 (PVI F_{6\%,10})$ $= 6,600(.5584)$ $= \underline{TK. 3685.44}$</p>
<p>b) $PV = \frac{FVn}{(1+\frac{i}{m})^{mn}} = \frac{9000}{(1+\frac{.12}{2})^{2 \times 8}} = \frac{9000}{2.54035} = \underline{TK. 3542.82}$</p>	<p>b) $PV = FVn (PVI F_{i/m,mn})$ $= 9,000 (PVI F_{12\%/2,2.8})$ $= 9,000 (PVIF 6\%, 16)$ $= \underline{TK. 3542.40}$</p>
<p>c) $PV = \frac{FVn}{(1+\frac{i}{m})^{mn}} = \frac{12000}{(1+\frac{.18}{4})^{4 \times 6}} = \frac{12000}{2.8760} = \underline{TK. 4172.46}$</p>	<p>c) $PV = FVn (PVI F_{i/m,mn})$ $= 12,000 (PVIF 4.5\%, 24)$ $= 12,000 (.3477)$ $= \underline{TK. 4172.40}$</p>
<p>d) $PV = \frac{FVn}{(1+\frac{i}{m})^{mn}} = \frac{15000}{(1+\frac{.12}{12})^{12 \times 3}} = \frac{15000}{1.4308} = \underline{TK. 10483.65}$</p>	<p>d) $PV = FVn (PVI F_{i/m,mn})$ $= 15,000 (PVIF 1\%, 36)$ $= 15,000 (.6989)$ $= \underline{TK. 10483.50}$</p>
	<p>e) $PV = \frac{FVn}{e^{i \times n}}$ $= \frac{18,000}{2.7183^{.12 \times 5}}$ $= \frac{18,000}{1.8221}$ $= \underline{TK. 9,878.71}$</p>

[Note – In the present Value Table, PVIF for 4.5% and 24 periods and 1% for 36 periods are not shown. Hence, in cases two cases PVIF has been found out by using the alternative formula which goes as under :

$$PVIF = \frac{1}{(1 + \frac{i}{m})^{mn}} - 1$$

Graphic View of the Discounting Process: Decrease

Figure–2 shows that Tk. 1 or any other sum to be received in the future diminishes as the time to receipt or the interest rate increases. An example shown in figure – 2 indicates the relationships among Present Value, Interest Rates and Time :



Solving Time and Interest Rates

In the determination of present values, future values and annuities, time factor and interest or discount factor have been worth-mentioning.

In the determination of present values and future values, time factor and interest or discount factor have been worth-mentioning. As for example, in determining future value, present value, time factor and interest factor must exist. On the other hand, in determining present value future value , time factor and interest factor must exist. It is evident that in each of these cases, the values of any three are given. The value of the fourth one can be found out. In such a context the necessity of determining the value of either interest (i) or period (n) has arisen.

In cases of Future Value (FV) and Present Value (PV)

Suppose, you can buy a security at a price of Tk. 78.35 that will pay you Tk. 100 after 5years. In this case, PV, FV, and n are given; we are to find out i, the interest rate you will earn on your investment.

Value of i is found out by applying the following formula :

$$FV_n = PV (1 + i)^n = PV (FVIFi, n)$$

$$\text{Hence, Tk. } 100 = \text{Tk. } 78.35 (1 + i)^5 = (FVIFi, n)$$

$$100$$

$$\text{or, } (1+i)^5 = \frac{100}{78.35} = 1.2763 = \text{FVIF}_{i, 5}$$

$$\text{or, } (1+i) = (1.2763)^{1/5} = 1.05$$

$$\text{or, } i = 1.05 - 1 = 0.05 = \underline{\underline{5\%}}$$

Solving for Period (n)

In cases of PV, FV and Annuities (ordinary and due), the period n can be found out if other elements of Time Value of Money viz.; PV, FV, Annuities and rate of interest/ discount (i) are given. The following paragraphs deal with the determination of period n.

Solving for period n in cases of FVs and PVs

Suppose you know that the investment in security will provide a return of 10% per year, that it will cost Taka 204.90 and that you will receive Tk. 300 at maturity. But, you do not know when the security matures. In this case you know PV, FV and i; but you are to know n, the number of periods. The solution is as under :

$$\text{We know that } FV_n = PV (1+i)^n = PV (\text{FVIF}_{i, n})$$

$$\text{or, } 300 = 204.90 (\text{FVIF}_{10\%, n})$$

$$300$$

$$\text{or, } \text{FVIF}_{10\%, n} = \frac{300}{204.90} = 1.4641$$

$$204.90$$

Now, let us look across the 10% column in Future Value Table until we find FVIF = 1.4641. This value is in Row 4, which indicates that it takes 4 years for Taka 204.90 to grow to Taka 300 at 10% interest rate.

Review Questions

A. Short Questions

1. What is 'Time Value of Money' ? Explain with an example.
2. Distinguish between future value and present value. Give example of each of them.
3. What is cash flow time line ? Explain with the help of a diagram.
4. What are the techniques of time value of money ? Explain with suitable examples.
5. Distinguish between compounding and discounting.
6. Define the following terms :
a) $FVIF_{i, n}$ b) $PVIF_{i, n}$ c) $FVIFA_{i, n}$ d) $PVIFA_{i, n}$.
7. What effects do (a) increasing rate of interest and (b) increasing time periods have on the
(i) Present value of a future sum and
(ii) Future Value of the present sum ? Why ?
8. Explain the significance of time value of money.
9. Distinguish between compound interest and simple interest with examples

B. Broad Questions

12. Discuss the tools and techniques of time value of money.
13. Explain the graphic views of the (i) compounding process and (ii) discounting process.

Review problems

Problem - 1

You have been awarded MBA degree from Bangladesh Open University just one month ago. You have applied for the post of Finance Manager in BRAC. As part of the BRAC's evaluation process, you have been asked to appear at a test that covers several financial analysis techniques. The first section of the test addresses time value of money analysis. See how would you do by answering the following questions :

- a) What is the future value of an initial investment of Taka 1,500 after 7 years if the investment earns 12% annual interest ?
- b) What is the present value of Taka 2,700 to be received in 5 years if the interest rate is 15% ?
- c) If a company's sales are growing at a rate of 20% annually, approximately how long will it take sales to triple ?
- d) What annual interest rate will cause Taka 2,200 to grow to Taka 7,700 in 5 years ?
- e) At what time periods will Taka 3,000 grow to Taka 9,000 if the rate of interest is 15% p. a. ?
- f) Which amount is worth more at 14% : Taka 1,000 in hand today or Taka 2,000 due in 6 years ?

Problem - 2

Find the amount to which Taka 1,500 will grow under each of the conditions :

- a) 12 percent compounded annually for 7 years;
- b) 12 percent compounded semiannually for 7 years;
- c) 12 percent compounded quarterly for 7 years;
- d) 12 percent compounded monthly for 7 years;
- e) 12 percent compounded daily for 7 years;
- f) 12 percent compounded continuously for 7 years.

Comment whether the growth will show an increasing or decreasing trend. Why ?

Problem - 3

Find the present value of Taka 5,000 due in the future under each of the conditions :

- a) 15 percent interest rate, compounded annually, discounted back 10 years;
- b) 15 percent interest rate, semiannually compounding, discounted back 10 years;
- c) 15 percent interest rate, quarterly compounding, discounted back 10 years;
- d) 15 percent interest rate, monthly compounding discounted back 5 years;
- e) 15 percent interest rate, daily compounding discounted back 6 years;
- f) 15 percent interest rate, continuously compounding discounted back 7 years.

Case Study :

A father is planning a savings program to put his daughter through university. His daughter is now 18 years old. He plans to enroll at the university in 5 years. Currently, the cost per year for everything – food, clothing, tuition fees, books, conveyance and so forth is Tk. 15,000, but a 5 percent inflation rate in these costs is forecasted. The daughter recently received Tk. 7,500 from her grand father's estate; this money which is invested in a mutual fund paying 8 percent interest compounded annually, will be used to help meet the cost of the daughter's education. The rest of the costs will be met by money the father will deposit in the savings account. He will 6 equal deposits to the account in each year from now until his daughter starts university. These deposits will begin today and will also earn 8 percent interest.

- a) What will be the present value of the cost of 5 years of education at the time the daughter becomes 24 ?
- b) What will be the value of Tk. 7,500 that the daughter received from her grand father when she starts university at the age 24 ?
- c) If the father is planning to make the first 6 deposits today, how large must each deposit be for him to be able to put his daughter through university ?

Lesson-4: Annuity : Time Value of Money

After attentively reading this lesson 4, you should be able –

- To know about concept of annuity;
- To know how to determine future value of an ordinary annuity and future value of an annuity due;
- To know how to determine of present value of an ordinary annuity and present value of an annuity due;
- To understand how to determine payments and
- To solve time and interest rates for annuities, ordinary and due.

Concept of Annuity

An annuity is a series of equal payments known as installments at fixed intervals for specified number of periods.

If the installment payments occur at the end of each period, as they typically do in business transactions, the annuity is known as ordinary/ deferred annuity. If installment payments are made at the beginning of each period, the annuity is called an annuity due.

Future value of an ordinary annuity depends on three things namely : (i) amount of PMT; (ii) rate of interest and (iii) period.

An annuity is a series of equal payments known as installments at fixed intervals for specified number of periods. As for instance Tk. 100 paid as an installment at the end of the each of the next five years is a five year annuity. The installment payments are symbolized as PMT and they can occur at either the beginning or the end of each year. If the installment payments occur at the end of each period, as they typically do in business transactions, the annuity is known as ordinary/ deferred annuity. If installment payments are made at the beginning of each period, the annuity is called an annuity due. Since ordinary annuities are more common in Finance, when the annuities are used in this book, you should assume that the installment payments occur at the end of each period values otherwise mentioned. Such annuities are closely related with PV and FV which are examined in the following sub-sections.

Future Value of an Ordinary Annuity

Future value of an ordinary annuity depends on three things namely : (i) amount of PMT; (ii) rate of interest and (iii) period. The more the amount of PMT, rate of interest and the period, the higher will be the amount of FV of an annuity. Let us take an example. If you deposit Taka 100 at the end of each of three years in a Savings A/C that pays 5% interest per year; how much will you have at the end of 3 years ? To answer this question, we must find out FV of an ordinary annuity (FVAn). Hence, FVAn represents the FV of an ordinary annuity over periods. Each payment is compounded out to the end of period n and the sum of the compounded payments is the FVAn.

There are two approaches of determining FVAn viz. (i) Equation Approach and (ii) Tabular Approach.

i) Under Equation Approach

ii) Under Tabular Approach

$$FVAn = PMT \sum_{t=0}^{n1} (1+i)^t$$

$$FVAn = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

$$= PMT(FVIFAi, n)$$

Explanation of FVIFAi,n :

The summation term in the brackets in the formula under Tabular Approach is called the Future Value Annuity Interest Factor for an annuity of n payments compounded at 1 percent of interest. In order to find out this interest factor, both n and I should be considered simultaneously in the Future Value Annuity Table.

Problem and Solution

Problem - 1

Find out the Future Values of the following ordinary annuities :

- (i) Taka 4,000 per year for 10 years at 12 percent;
- (ii) Taka 2,000 per year for 5 years at 10 percent;
- (iii) Taka 1,000 per year for 6 years at 0 percent.

Solution (After tabular approach)

(i) $FVAn = PMT (FVIFAi, n)$ $= 4,000 (FVIFA12\%, 10)$ $= 4,000 (17.549)$ $= Tk. \underline{70,196}$	(ii) $FVAn = PMT (FVIFAi, n)$ $= 2,000 (FVIFA10\%, 5)$ $= 2,000 (6.1051)$ $= Tk. \underline{1,2210.20}$
---	--

(iii) $FVAn = PMT (FVIFAi, n)$
 $= 1,000 (FVIFA0\%, 6)$
 $= 1,000 (6)$
 $= Tk. \underline{6,000}$

Future Value of an Annuity Due

Like future value of an ordinary annuity, future value of an annuity due also depends on the :

- (i) amount of payments; (ii) rate of interest and (iii) number of periods.
- The more the amount of PMT, rate of interest and the number of periods; the higher will be future value of annuity due. Had there been Tk. 100 payments in the previous example being made at the beginning of each year, the annuity would have been known as an annuity due.

Future value of an annuity due FVA (DUE) can also be found out in two approaches viz. : (i) Equation Approach and (ii) Tabular Approach.

Had there been Tk. 100 payments in the previous example being made at the beginning of each year, the annuity would have been known as an annuity due.

Under Equation Approach

$$FVA(DUE) = PMT \left[\left\{ \frac{(1+i)^n - 1}{i} \right\} \times (1+i) \right]$$

Under Tabular Approach

$$FVA(DUE) = PMT \left[\{FVIFA_{i,n}\} \times (1+i) \right]$$

Future value interest factor annuity (DUE) for n periods at I interest percent can be found from the Future Value Annuity Table, considering n periods and I interest rates.

Problem and Solution

Problem – 2

Find out the future value of the following annuities due -

- (a) Tk. 3,000 per year for 8 years at 8%;
- (b) Tk. 5,000 per year for 10 years at 12% and
- (c) Tk. 2,000 per year for 7 years at 0%.

Solution

$$FVA (DUE) = PMT [(FVIFA_{i, n}) (1 + i)]$$

$$\begin{aligned} \text{(a) FVA (DUE)} &= 3,000 [(FVIFA_{8\%, 8}) (1+.08)] \\ &= 3,000 [(10.637) (1.08)] \\ &= \underline{\text{Tk. 34,463.88}} \end{aligned}$$

$$\begin{aligned} \text{(b) FVA (DUE)} &= PMT [(FVIFA_{i, n}) (1 + i)] \\ &= 5,000 [(17.549) (1.12)] \\ &= \underline{\text{Tk. 98,274.40}} \end{aligned}$$

$$\begin{aligned} \text{(c) FVA (DUE)} &= PMT [(FVIFA_{i, n}) (1 + i)] \\ &= 2,000 [(FVIFA_{0\%, 7})] \\ &= \underline{\text{Tk. 14,000}} \end{aligned}$$

Present Value of an Ordinary Annuity

Present value of an ordinary annuity refers to the value today of a future ordinary annuity.

Present value of an ordinary annuity refers to the value today of a future ordinary annuity. Suppose you are offered the following alternatives : (i) a three year annuity with payments of Tk. 100 at the end of each of the 3 years and (ii) a lump-sum payment today. Now, the question is : How large must the lump-sum payment today be to make it equivalent to the annuity ? To answer this question, we must find out the present value of

an ordinary annuity (PVAn). Each of the payment is to be discounted and the sum of the discounted payments is the PVAn.

There are two approaches of finding out PVAn viz.: (i) Equation Approach and (ii) Tabular Approach.

<u>Under Equation Approach</u>	<u>Under Tabular Approach</u>
$PVAn = PMT \left[\sum_{t=1}^n \frac{1}{(1+i)^t} \right]$	$PVAn = PMT (PVIFA_{i,n})$

PVIFA refers to the summation term in the bracket in this Equation is called the Present Value Interest Factor Annuity. It is the present value interest factor for an annuity of n periods, discounted at I interest percent. In order to find out this interest factor, Present Value Annuity Table should be consulted considering n periods and discounted I interest factor. The present value of an annuity depends on : (i) amount of PMT; (ii) n periods and (iii) rate of discount i. The more the amount of PMT, n periods and rate of discount, the higher will be the amount of annuity and vice-versa.

It is the present value interest factor for an annuity of n periods, discounted at I interest percent.

Problem And Solution

Problem - 3

Find out the present values of the following ordinary annuities :

- a) Taka 2,500 for 10 years at 12 percent;
- b) Taka 4,500 for 12 years at 10 percent and
- c) Taka 6,000 for 8 years at 0 percent.

Solution

a) $PVAn = PMT (PVIFA_{i,n})$ $= \text{Tk. } 2,500 (PVIFA_{12\%, 10})$ $= \text{Tk. } 2,500 (5.6502)$ $= \text{Tk. } \underline{14,125.50}$	b) $PVAn = PMT (PVIFA_{i,n})$ $= \text{Tk. } 4,500 (PVIFA_{10\%, 12})$ $= \text{Tk. } 4,500 (6.8137)$ $= \text{Tk. } \underline{30,661.65}$
--	--

c) $PVAn = PMT (PVIFA_{i,n})$
 $= \text{Tk. } PMT (PVIFA_{0\%, 8})$
 $= \text{Tk. } 6,000 (8)$
 $= \text{Tk. } \underline{48,000}$

Present Value of an Annuity Due

In the previous example cited in 3.3.3, had the three Tk. 100 payments been made at the beginning of each of the three years; the annuity would

In the previous example cited, had the Tk. 100 payments been made at the beginning of each of the three years; the annuity would have been an annuity

have been an annuity due. Like the present value of an ordinary annuity; present value of an annuity due can be found out on the basis of the amount of payment, n periods and present discount rate. More the amount of PMT, n periods and i percent of discount; the higher will be the annuity due and vice-versa.

Present value of an annuity due can also be measured by two approaches viz. : (i) Equation Approach and (ii) Tabular Approach.

(i) Equation Approach

$$PVAn(DUE) = PMT \left[\sum_{t=1}^n \frac{1}{(1+i)^t} \right] \times (1+i)$$

The present interest factor for an annuity due – PVIFAn (DUE)I, n is equal to PVIFAi, n X (1 +

i). Such interest factor would be found out from the Present Value Annuity Table, considering n periods and I rate of discount.

(ii) Tabular Approach

$$PVAn (DUE) = PMT [(PVIFAi, n) (1 + i)]$$

Problem and Solution

Problem - 4

Find the present value of the following annuities; if the PMT occur at the beginning of the year i.e. annuities due :

- a) Taka 7,500 for 9 years at 14 percent;
- b) Taka 10,000 for 5 years at 9 percent and
- c) Taka 6,600 for 7 years at 0 percent.

Solution

$$\begin{aligned} PVA (DUE) &= PMT (PVIFAi, n) \times (1 + i) \\ &= Tk. 7,500 (PVIFA14\%, 9) \times (1 + 0.14) \\ &= Tk. 7,500 (4.9464) \times (1.14) \\ &= Tk. \underline{42291.72} \end{aligned}$$

$$\begin{aligned} b) PVA (DUE) &= PMT (PVIFAi, n) \times (1 + i) \\ &= Tk. 10,000 (PVIFA9\%, 5) \times (1 + 0.09) \\ &= Tk. 10,000 (3.8897) \times (1.09) \\ &= Tk. \underline{42,397.73} \end{aligned}$$

$$\begin{aligned} c) PVA (DUE) &= PMT (PVIFAi, n) \times (1 + i) \\ &= Tk. 6,600 (PVIFA0\%, 7) \times (1 + 0) \\ &= Tk. 6,600(7) \times (1) \\ &= Tk. \underline{46,200} \end{aligned}$$

Determination of Payments (PMT)

In this sub-section, we shall examine how payments (PMT) are determined in cases of both types of annuities viz. ordinary annuity and annuity due; where the values of annuities, rate of interest i and period n are given.

a) Determination of PMT in case of Ordinary annuity

Suppose you have borrowed Taka 18,000 from a bank with 12% interest for a period of 15 years. What is the annual interest payment if the payments are to be made at the end of each year ?

This is a case of ordinary annuity. So, $PVAn = PMT (PVIFA_{i, n})$

$$18,000 = PMT (PVIFA_{12\%, 15})$$

$$18,000 = PMT (6.8109)$$

$$\underline{18,000} = PMT$$

$$6.8109 \quad PMT = \text{Taka } \underline{2642.82}$$

b) Determination of PMT in case of Annuity Due

Suppose you have borrowed house building loan of Taka 10 lacs from HBFC with 10% interest for a period of 20 years. What is the annual interest payment if the payments are to be made at the beginning of each year ?

This is a case of annuity due. So, $PVAn (DUE) = PMT (PVIFA_{i, n})$

$$(1 + i) 10,00,000 = PMT (PVIFA_{10\%, 20}) (1 + 0.10)$$

$$10,00,000 = 1.10 PMT (8.5136)$$

$$PMT = \frac{10,00,000}{1.10 \times 8.5136} = \frac{10,00,000}{9.365} = \text{Tk. } \underline{1,06,780.57}$$

Solving Time and Interest Rates

In the determination of annuities, time factor and interest or discount factor have been worth-mentioning. While determining annuities, either ordinary or due; payment, time factor and interest factor must exist. It is evident that in each of these cases, the values of any three are given. The value of the fourth one can be found out. In such a context the necessity of determining the value of either interest (i) or period (n) has arisen.

In case of Annuities (Ordinary and Due)

In the previous problems the FVs and PVs of ordinary annuity as well as annuity due have been found out where PMT, i and n are given. But, here we are interested to determine i where FVs or PVs, PMT and n are given. For the purpose of determining i , the same formula given under Tabulation Approach while calculating FVs and PVs in cases of ordinary

In the determination of present values, future values and annuities, time factor and interest or discount factor have been worth-mentioning.

annuity and annuity due need to be followed. The following problem deals with the calculation of i .

Problem and Solution

Problem - 5

Find out the interest rate (i) in the following cases :

- You borrow Taka 9,000 and promise to make equal payments of Taka 2,684.80 at the end of each year for 5 years;
- You borrow Taka 13,250 and promise to make equal payments of Taka 2,640.07 at the beginning of each year for 10 years.

Solution

This problem relates to ordinary annuity; since payments are made at the end of the year. So, the formula for ordinary annuity will be followed which is given as :

$$PVAn = PMT (PVIFA_i, n)$$

$$\text{or, } 9,000 = 2,684.80 (PVIFA_i, 5)$$

$$\text{Hence, } PVIFA = \frac{9,000}{2,684.80} = 3.3522$$

In Present Value Annuity Table, let us look across the period (n) 5 row until we find $PVIFA = 3.3522$. This value lies in the 15% columns; so the interest rate at which a five year 2,684.80 annuity has a PV of Taka 9,000 is 15 percent.

b) This problem relates to annuity due; since payments are made at the beginning of the year. So, the formula for annuity due will be applied which is as under :

$$PVAn (DUE) = PMT [(PVIFA_i, n) (1 + i)]$$

$$\text{or, } 13,250 = 2,640.07 [(PVIFA_i, 10) (1 + i)]$$

$$\text{Hence, } PVIFA_{i, 10} = \frac{13,250}{2,640.07} = 5.0188 (1 + i)$$

In the Present Value Annuity Table, let us look across the period (n) 10 row until we find $PVIFA = 5.0188$. This value lies in 15% column; so the interest rate at which a ten-year 2,640.07 annuity has a PV of Taka 13,250 is 15 – 0.15 percent i.e. 14.85%.

b) Solving period n in cases of Annuities, Ordinary and Due

In case of either ordinary annuity or annuity due, period n can be found out if values of ordinary annuity or annuity due, PMT and i are given. The following examples will clear the matter.

Examples

- Suppose you borrow Taka 15,000 and promise to make equal installment payments of Taka 2,604.62 at the end each of the requisite

years at 10 percent.. In this case, you know the value of ordinary annuity, PMT and i; you are to determine period n. The solution goes as follows :

$$PVAn = PMT (PVIFAi, n)$$

$$\text{or, } 15,000 = 2,604.62 (PVIFA10\%, n)$$

$$PVIFA10\%, n = \frac{15,000}{2,604.62} = 5.759$$

In Present Value Annuity Table, let us look across the 10% column until we find PVIFA = 5.759. This value lies in Row 9, which indicates that it takes 9 years for Taka 2,604.62 to grow to Taka 15,000 at 10% interest rate.

b) Suppose you borrow Taka10,000 and promise to make equal installment payments of Taka 2,054.06 at the beginning of each of the requisite years of 10 percent. In this case, you know the value of annuity due, PMT and i; you are to find out period n. The solution goes as under:

$$PVAn (DUE) = PMT (PVIFAi, n) (1 + i)$$

$$\text{or, } 10,000 = 2,054.06 (PVIFA10\%, n) (1 + 0.12)$$

$$\text{or, } PVIFAn10\%, n = \frac{10,000}{2,054.06} (1 + 0.10)$$

$$= 4.8684 (1 + 0.10) = 5.3552$$

In the Present Value Annuity Table, let us look across the 10% column until we find PVIFA = 5.3552. This value lies around Row 7, which indicates that it takes around 7 years for Taka 2,054.06 to grow to Taka 10,000 at 10% interest rate.

Review Questions

A. Short Questions

1. Define ordinary annuity and annuity due. How are they determined? Explain.
2. For a given interest rate and given number of years, is the factor for the sum of an annuity larger or smaller than the interest factor for the present value of the annuity ?
3. How would you determine payments in case of ordinary annuity and annuity due ?

B. Broad Questions

4. What are (i) Annuity (Ordinary) and (ii) Annuity (Due) ? Explain their relationship with the FV and PV.
5. How would you solve for time and interest rates in cases of (i) ordinary annuity and (ii) annuity due ?
6. How would you determine future value and present value in cases of (i) ordinary annuity and (ii) annuity due ?

Review Problems

Problem - 1

Find the present value of Taka 5,000 due in the future in case of annuity due and ordinary annuity under the following conditions :

- a) 15 percent interest rate, compounded annually, discounted back 10 years;
- b) 15 percent interest rate, semiannually compounding, discounted back 10 years;
- c) 15 percent interest rate, quarterly compounding, discounted back 10 years;
- d) 15 percent interest rate, monthly compounding discounted back 5 years;
- e) 15 percent interest rate, daily compounding discounted back 6 years;
- f) 15 percent interest rate, continuously compounding discounted back 7 years.

Problem - 2

To help you reaching your Tk. 10,000 goal, your mother offers to give you Tk. 4,000 on January 1, 2001. You will get a part time job and make 6 additional payments of equal amount each of 6 months thereafter. If all these money is deposited in bank that pays 12 percent, compounded semiannually, how large must each of the 6 payments be ?

Problem - 3

Find the future value of Taka 15,000 in case of annuity due and ordinary annuity under the following conditions :

- a) 12 percent interest rate, compounded annually, discounted back 10 years;

- b) 12 percent interest rate, semiannually compounding, discounted back 10 years;
- c) 12 percent interest rate, quarterly compounding, discounted back 10 years;
- d) 12 percent interest rate, monthly compounding discounted back 5 years;
- e) 12 percent interest rate, daily compounding discounted back 6 years;
- f) 12 percent interest rate, continuously compounding discounted back 7 years.

Lesson–5: Valuation of Long-term Securities

After successfully completing this lesson 5, you should be able:

- To form a clear concept on the fundamental valuation Concepts and
- To know about the techniques of valuation of long-term securities viz., bond and stock.

Fundamental Valuation Concepts

In addition to the present value concept as examined in the previous lesson, there are many other concepts of value, used for different purposes. The following paragraphs examine some of the important concepts of value as used in finance and financial management.

Book Value

Book value reflects historical cost, rather than value. It is an accounting concept; but not financial one. Assets are recorded at historical cost and they are depreciated over years. Therefore, book value may include intangible assets at acquisition cost minus amortized value. The book value per share is found out as net worth divided by the number of shares outstanding.

Replacement Value

Replacement value is the amount that a firm would be required to spend if it were to replace its existing assets in the current condition. It is difficult to find cost of assets currently being used by the firm. It is likely to ignore the benefits of intangibles and the utility of existing assets.

Liquidation Value

Liquidation value is the amount that a firm could realize if it sold its assets after having liquidated or terminated its business. It would not include the value of intangibles since the operations of the firm are assumed to cease.

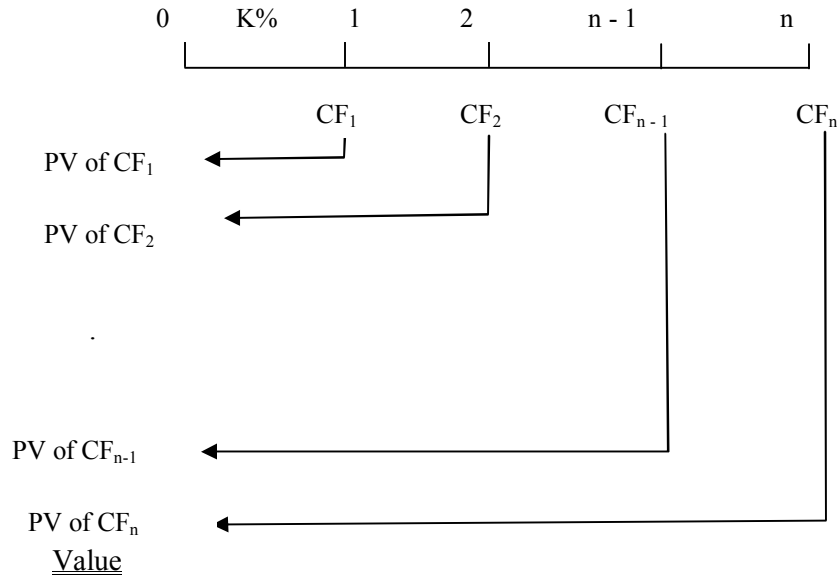
Going Concern Value

It is the amount that a firm could realize if it sold its business as an operating business. Such value would always be higher than the liquidation value.

Market Value

Market value of an asset or security is the current price at which the asset or the security is being sold or bought in the market. Market value per share is expected to be higher than the book value per share. A number of factors influence the market value per share; and, therefore, it shows wide fluctuations.

After learning about the Time Value of Money one should realize that the value of anything, whether it is a financial asset like bond or stock or a real asset like plant and machinery, building, is based on the present value of the cash flows the asset is expected to produce in the future. On a cash flow time line, value can be exhibited as under :



Therefore, the value of any asset can be expressed in general form in the following equation:

$$V = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \dots + \frac{CF_n}{(1+k)^n}$$

Where - CF₁ = Expected cash flows;

k = Rate of required return.

n= No. of year/period

According to the above Equation, the value of an asset is affected by expected cash flows (CF₁) and the return required by the investors (k). As you can see, the higher the expected CF, the greater the asset's value; also the lower the required return, the greater the asset's value.

Valuation of Bonds

A bond or debenture is a long-term debt instrument. Bonds issued by the government or the public sector companies in Bangladesh are generally secured. But, the bonds issued by the private sector companies may be either secured or unsecured. A bond possess some features of which the following are the main :

A bond or debenture is a long-term debt instrument.

i) **Face value** : It is also called per value. A bond is generally issued at a par value and interest is paid on face value.

ii) Interest rate : It is fixed and known to bondholders. It is also called coupon rate. It is a rate mentioned on the certificate on the bond.

iii) Maturity : A bond issued for a specified period of time. It is repaid on maturity date.

iv) Redemption Value : The value which a bondholder will get on maturity is called redemption value. A bond may be redeemed at par or at a premium (more than par value) or at a discount (less than par value).

v) Market Value : A bond may be traded in a stock exchange. The price at which currently sold or bought is called the market value. Market value may be different from par value or redemption value.

Bonds may be of two types viz., (a) bonds with maturity and (b) bonds without maturity.

Perpetual bonds:

When a bond or a debenture has a finite maturity, to determine its par value, we shall consider annual interest payments plus its terminal or maturity value.

(a) Bond with a Maturity Period : When a bond or a debenture has a finite maturity, to determine its par value, we shall consider annual interest payments plus its terminal or maturity value. Using the present value concept, the discounted value of these flows will be calculated. By comparing the PV of a bond with its market value; it can be determined if the bond is overvalued or undervalued.

Bond value (V_d) can be found out using the following formula :

$$V_d = \sum_{t=1}^N \frac{INT}{(1 + kd)^t} + \frac{M}{(1 + kd)^N}$$

$$= INT (PVIFA_{kd,N}) + M(PVIF_{kd,N})$$

- Where, INT = Taka Interest
- kd = Average rate of return
- M = Par value
- N = No. of years before bond matures

Although some bonds pay interest annually, most bonds actually pay interest semiannually. To evaluate, semiannual payment bonds, we must modify the valuation equation.

Bonds with Semiannual Compounding

In this case we can follow the following formula:

$$V_d = \sum_{t=1}^{2N} \frac{\left(\frac{INT}{2}\right)}{\left(1 + \frac{kd}{2}\right)^t} + \frac{M}{\left(1 + \frac{kd}{2}\right)^{2N}}$$

$$= \frac{INT}{2} (PVIFA_{\frac{kd}{2}, 2N}) + M(PVIF_{\frac{kd}{2}, 2N})$$

(b) Perpetual Bond

Bonds which will never mature are known as perpetual bonds. Such bonds are rarely found in practice. Since perpetual bonds have no maturity; so there is no terminal value. Therefore, the value of the bonds would simply be discounted value of the infinite stream of interest flows. Therefore, to find value of a perpetual bond is too easy which goes as follows :

Bonds which will never mature are known as perpetual bonds.

$$V_d = \frac{INT}{kd}$$

The value of a bond depends on the interest rate risks. As the interest rate changes, the value of a bond also changes. There is an inverse relationship between the value of a bond and the interest rate. The value will decline when the interest rate rises and vice-versa. Thus, the investors investing their funds in bonds are exposed to risk from increasing or falling interest rates. The intensity of interest rate risk would be higher on bonds with long maturities than those in short maturities.

There is an inverse relationship between the value of a bond and the interest rate.

Problems and Solutions

Problem - 1

The Beta Corporation issued a new series of bonds on January 1, 1980. The bonds were sold at par value of Taka 1,000, have a 12 percent coupon and would mature on December 31, 2009. Coupon payments are made semiannually i.e. on June 30 and December 31.

What was the price of the bond on January 1, 1985 assuming that the level of interest rates had fallen to 10 percent

Solution

$$\begin{aligned} V_d &= \sum_{t=1}^{2N} \frac{\left(\frac{INT}{2}\right)}{\left(1 + \frac{kd}{2}\right)^t} + \frac{M}{\left(1 + \frac{kd}{2}\right)^{2N}} \\ &= \frac{INT}{2} (PVIFA_{\frac{kd}{2}} \cdot 2N) + M(PVIF_{\frac{kd}{2}} \cdot 2N) \\ &= \frac{120}{2} (PVIFA_{5.50}) + M(PVIF_{5.50}) \\ &= 60(18.2559) + 1000(0.0872) \\ &= Tk.1,095.35 + 87.20 = Tk.1,182.55 \end{aligned}$$

Problem - 2

The bonds of Leema Corporation are perpetuities with a 10% coupon. Bonds of such type are currently yield 8% and their par value is Tk. 1,000.

- a) What is the price of the coupon ?
- b) Suppose interest rate levels rise to the point where such bonds now yield 12%. What would be the price of bonds ?
- c) At what price would the Leema Corporation sell if the yield on those bonds was 10% ? and
- d) How would your answers to parts a, b and c change if the bonds were not perpetuities but had a maturity of 20 years ?

Solution

a) In case of perpetual bonds :

$$V_d = \frac{\text{INT}}{K_d} = \frac{10\% \text{ of } 1,000}{0.08} = \frac{100}{0.08} = \text{Tk. } 1,250$$

b) In case of perpetual bonds :

$$V_d = \frac{\text{INT}}{K_d} = \frac{100}{0.12} = \text{Tk. } 833.33$$

c) In case of perpetual bonds :

$$V_d = \frac{\text{INT}}{K_d} = \frac{100}{0.10} = \text{Tk. } 1,000$$

d) In case of Maturity Bonds :

i) At 8% kd, $V_d = \text{INT} (\text{PVIFA}_{k_d, N}) + M(\text{PVIF}_{k_d, N})$
 $= 100 (\text{PVIFA}_{8\%, 20}) + 1,000 (\text{PVIF}_{8\%, 20})$
 $= (100 \times 9.8181) + (1,000 \times 0.2145)$
 $= 981.81 + 214.50 = \text{Tk. } \underline{1,196.31}$

ii) At 12% kd, $V_d = \text{INT} (\text{PVIFA}_{k_d, N}) + M(\text{PVIF}_{k_d, N})$
 $= 100 (\text{PVIFA}_{12\%, 20}) + 1,000 (\text{PVIF}_{12\%, 20})$
 $= (100 \times 7.4694) + (1,000 \times 0.1037)$
 $= 746.94 + 103.70 = \text{Tk. } \underline{850.64}$

iii) At 10% kd, $V_d = \text{INT} (\text{PVIFA}_{k_d, N}) + M(\text{PVIF}_{k_d, N})$
 $= 100 (\text{PVIFA}_{10\%, 20}) + 1,000 (\text{PVIF}_{10\%, 20})$
 $= (100 \times 8.5136) + (1,000 \times 0.1486)$
 $= 851.36 + 148.60 = \text{Tk. } 999.96 \text{ or, Tk. } \underline{1,000}$

The preferred stockholders are generally paid dividend at the fixed rate every year. But, stockholders' dividend varies and in case of losses, no dividend is paid to them.

Valuation of Stocks

Each company or corporation issues at least one type of stock/ share or equity, called common stock. Some of them issue more than one type of common stock and some issue preferred stock in addition to common stock. As the name implies in case of preferred stocks, the holders have preference over the common stock holders in respect of profits, proceeds at the time of liquidation etc. The preferred stockholders are generally paid dividend at the fixed rate every year. But, the common

stockholders' dividend varies and in case of losses, no dividend is paid to them. For all these reasons, the valuation processes of common stocks and preferred stocks differ. The following sub-sections deal with each of them.

Valuation of Common Stocks

Stock prices are determined at the equation similar to the bond valuation equation, explained in earlier. Therefore, to compute the value of stock, the following equation needs to be used :

$$V_S = \sum_{t=1}^{\infty} \frac{\hat{D}_1}{(1 + k_s)^t}$$

The valuation of stocks may relate to the three situations as follows : (i) valuating stocks with zero growth of dividend; (ii) valuating stocks with constant growth and (iii) valuating stocks with non-constant growth. The following paragraphs deal with them.

(a) Valuing Stock with Zero Growth

In case of stocks when dividends are not expected to grow at all; this is known as zero growth stocks. In these cases, dividends are expected to remain the same every year. Such stocks can be valued by applying the following formula :

$$\hat{P}_0 = \frac{D}{k_s}$$

A zero-growth stock is, therefore, a perpetuity.

(b) Valuing Stock with Constant Growth

In cases of stocks where dividends are expected to grow at a normal or constant rate, this is known as constant growth stocks. Such stocks can be valued by using the following formula :

$$\hat{P}_0 = \frac{\hat{D}_1}{K_s - g} = \frac{D_0(1 + g)}{K_s - g}$$

Where : \hat{P}_0 = Current stock price,

D_0 = Last dividend.

K_s = Required rate of return on stock

g = Growth rate in dividend

The valuation of stocks may relate to the three situations as follows : (i) valuating stocks with zero growth of dividend; (ii) valuating stocks with constant growth and (iii) valuating stocks with non-constant growth.

In cases of stocks where dividends are expected to grow at a normal or constant rate, this is known as constant growth stocks.

(c) Valuing Stocks with Non-constant Growth

In the case of stocks where dividends grow firstly in some years; but in other years dividends grow slowly or constantly; thus, it is known as non-constant growth.

In the case of stocks where dividends grow firstly in some years; but in other years dividends grow slowly or constantly; thus, it is known as non-constant growth. Such stocks can be valued by applying the following formula :

$$P\hat{V}\hat{D} = \hat{D}_1 + \hat{D}_2 + \dots + \hat{D}_n$$

Where, $\hat{D}_1 = \text{Last Dividend} \times g$

$$\hat{D}_2 = \hat{D}_1 \times g$$

$$\hat{D}_n = \hat{D}_{n-1} \times g$$

$$\hat{P}_0 = P\hat{V}\hat{D} + P\hat{V}\hat{P}_n$$

Problems and Solutions

Problem - 1

Batex Company's current stock price is Tk. 36 and its last dividend was Tk. 2.40 per stock. In view of its strong financial position and its consequent low risk, its required rate of return, (ks) is only 12%. If dividends are expected to grow at a constant rate, g, in the future and its ks is expected to remain at 12%, what is the company's expected stock price, 5 year from now ?

Solution

$$D_0 (1 + g)$$

We know that $\hat{P}_0 = \frac{\dots}{(k-g)}$

$$\text{Tk. } 2.40 (1 + g)$$

$$\text{or, } 36 = \frac{\dots}{0.12 - g}$$

$$\text{or, } 0.12(36) - 36g = 2.40 + 2.40g$$

$$\text{or, } 4.32 - 2.40 = 36g + 2.40g$$

$$\text{or, } g = 0.05$$

$$\text{or, } g = 5\%$$

Hence, stock price after 5 years from now :

$$\hat{P}_5 = \frac{D_0(1+g)^6}{K_s - g} = \hat{P}_5 = \begin{matrix} \text{PV of } \hat{P}_0 \text{ at growth rate} \\ = \text{PV of Tk. } 36 (1+0.05)^5 \\ = \text{Tk. } 45.95 \end{matrix}$$

Problem - 2

Computer Tech. Inc. is experiencing a period of rapid growth. Earnings and dividends are expected to grow at a rate of 15% during the first 3 years, at 13% in the 4th year and at a constant rate of 6% thereafter. Its last dividend was Tk. 1.15 and the required rate of return on the stock is 12%. Calculate the value of stock today.

Solution

(i) Calculation of PV of dividends paid during the growth period :

$$\begin{aligned} \hat{D}_1 &= \text{Tk. } 1.15 (1.15) = \text{Tk. } 1.3225 \\ \hat{D}_2 &= \text{Tk. } 1.3225 (1.15) = \text{Tk. } 1.5209 \\ \hat{D}_3 &= \text{Tk. } 1.5209 (1.15) = \text{Tk. } 1.7186 \\ PV \hat{D} &= \text{Tk. } 1.3225 (0.8929) + \text{Tk. } 1.5209 (0.7972) + \\ &\quad \text{Tk. } 1.7186 (0.7118) \\ &= \text{Tk. } 1.1809 + \text{Tk. } 1.2125 + \text{Tk. } 1.2233 \\ &= \text{Tk. } 3.6167 = \text{Tk. } \underline{3.62} \end{aligned}$$

(ii) PV of stock price at the end of the year 3 :

$$\begin{aligned} \hat{P}_3 &= \frac{\hat{D}_3(1+g)}{K_s - g} = \frac{1.7186(1+0.06)}{0.12 - 0.06} = \text{Tk. } 30.36 \\ PV \hat{P}_3 &= \text{Tk. } 30.36(0.7118) = \underline{\underline{\text{Tk. } 21.61}} \end{aligned}$$

$$\begin{aligned} \text{(iii) Value of stock today} &= \sum (PVD + PV\hat{P}_3) \\ &= \text{Tk. } 3.62 + 21.61 \\ &= \text{Tk. } \underline{\underline{25.23}} \end{aligned}$$

Review Questions

A. Short Questions

1. What is concept on “Fundamental Valuation” ? Explain.
2. In which equation form, the value of any asset can be expressed? Explain how cash flows and the required return of an investor affect the value of an asset.
3. What are the main features of a bond ? Explain.
4. Examine the relationship between bond maturity and interest rate risk.
5. Why does the valuation process of common stock and preferred stock differ ?

B. Broad Questions:

6. What are the various types of a bond ? How are their value determined ?
7. How common stocks are valued with the following situations :
(i) Zero growth; (ii) Constant growth and
(iii) Non-constant growth ?

Review Problems

Problem - 1

Following are the cash inflows of two projects A and B.

Different Situation	Expected Income		Probability
	Project - A	Project - B	
1	5,000	0	0.05
2	8,000	5,000	0.20
3	10,000	10,000	0.50
4	12,000	15,000	0.20
5	15,000	20,000	0.05

Required :

- a) The expected value of both the projects A and B and
- b) Standard deviation of the value of the projects.

Problem – 2

Suppose Ford Motor Company sold an issue of bonds with a ten year maturity a \$1,000 par value, a ten percent coupon rate and semiannual interest payments,

- a) Two years after the bonds were issued, the going rate of interest on bond such as these fell to six percent. At what price would the bond sell?
- b) Suppose that, two years after the initial offering, the going interest rate had risen to 12 percent. At what price would the bond sell?
- c) Suppose that the conditions in part a existed – that is, interest rates fall to six percent two years after the issue date. Suppose further that the interest rate remained at six percent for the next eight years. Describe what would happen to the piece of the Ford Motor Company bonds over time.

Problem - 3

The bonds of the Lange Corporation are perpetuities with a ten percent coupon. Bonds of this type currently yield eight percent, and their par value is \$1,000.

- a) What is the price of Lange bonds?
- b) Suppose interest rates levels rise to the point where such bonds now yield 12 percent. What would be the price of the Lange bonds?
- c) At what price would the Lange bonds sell if the yield on these bonds was ten percent?
- d) How would your answers to parts a, b, and c change if the bonds were not perpetuities but had a maturity of 20 years.

Problem - 4

The Desreumaux Company has two bond issues outstanding. Both bonds pay \$100 annual interest plus \$1000 at maturity. Bond L has a maturity of 15 years and Bond S a maturity of 1 year.

- a) What will be the value of each of these bonds when the going rate of interest is (i) 5 percent, (ii) 8 percent, and (iii) 12 percent? Assume that there is only one more interest payment to be made on Bond S.
- b) Why does the longer term (15 years) bond fluctuate more when interest rate change than does the shorter – term (1 year) bond?

Problem - 5

Microtech Corporation is expanding rapidly, and it currently needs to retain all of its earnings; hence, it does not pay any dividends. However, investors expect Microtech to begin paying dividends, with the 1st dividend of \$1 coming three years from today. The dividend should grow rapidly – at a rate of 50 percent per year – during Years 4 and 5. After Year 5, the company should grow at a constant rate of eight percent per year. If the required rate of return on the stock is 15 percent, what is the value of the stock today?

Lesson-6: Measurement of Returns from Long-term Securities

After successfully completing this lesson 6, you should be able :

- To study the techniques of measuring returns from bonds;
- To study the techniques of measuring stock return and
- To examine the criteria used in measuring financial risk involved in these returns.

Techniques of Measuring Returns

Internal Rate of Return or yield for an investment is the discount rate that equates the present value of the expected cash outflows with the present value of the expected cash inflows.

We know that long-term securities consist of long-term bond and stock, common and preferred. The return from such securities is also known as yield, which refers to the internal rate of return (IRR). IRR or yield for an investment is the discount rate that equates the present value of the expected cash outflows with the present value of the expected cash inflows. IRR can be found out by applying the following equation :

$$A_0 = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \dots + \frac{A_n}{(1+r)^n}$$

Where : A_0 = Initial cash out flows or outlays.

A_1, A_2, \dots, A_n = Cash in flows.

r = Required rate of return.

n = No. of years

Against the above discussion, let us discuss the technique as used in measuring returns from Bonds and Stocks. The following sub-sections deal with them.

Bond Returns

Bond return or bond yield is simply a bond's internal rate of return (IRR).

Bond return or bond yield is simply a bond's internal rate of return (IRR). While valuing bonds, we have mentioned that bonds are of two types namely : (a) Maturity Bond and (ii) Perpetual Bond. So, bond return differs in cases of these two types of bonds; which are explained as follows:

(a) Returns from Maturity Bonds

In case of maturity bonds, yield to maturity (YTM) is applicable which is the average rate of return earned on a bond, if it is held to maturity.

In case of maturity bonds, yield to maturity (YTM) is applicable which is the average rate of return earned on a bond, if it is held to maturity. In case of maturity bonds, there are two discounts or coupon bonds viz., (i) Pure discount (zero coupon bonds) and (ii) Coupon bonds.

- Zero Coupon Bonds:** It is one where the issuer promises to make a single payment at a specified future date. The single payment is the same as the face value of the bond. In case of zero coupon bonds, YTM is found out as follows :

$$P = \frac{\text{Face Value}}{(1 + r/2)^{2n}}$$

Where : P = Present market price of the bond;

r = Yield to Maturity (YTM);

n = Maturity.

(ii) In case of Coupon Bonds

To determine the YTM (r) in case of the coupon bond, the following equation is to be used :

$$YTM = \frac{INT + \frac{M - Vd}{N}}{\left[\frac{2(Vd) + M}{3} \right]}$$

Where : M = Par Value of Bonds;

Vd = Market value of bond and

N = No. of years,

b) Returns from Perpetual Bonds

With a perpetuity, a fixed cash inflow is expected at equal intervals for ever. In case of perpetual bonds, return (r) is calculated as follows :

$$r = \frac{A^*}{A_0}$$

Where : A* = Fixed annual interest payment and

A₀ = Market price of the bond.

Problems and Solutions

Problem - 1

Find IRR in cases of the following investments:

- An investment of Tk. 10,000 today will return Tk. 2,000 at the end of 10 years.
- An investment of Tk 1,000 today will return Tk 500 at the end of each of the next 3 years.
- An investment of Tk. 1,000 will return Tk. 60 per year forever.

Solution

a) Cash outflow (Investment) = Tk. 10,000

Cash inflows at the end of 10 years = Tk. 2,000

$$\text{PVAF} = \frac{10,000}{2,000} = 5.000 \text{ (It is also known as pay back period (PB))}$$

Looking at the Present Value Annuity Table across 10 years row, this factor 5.000 is found within 15% and 16% columns. Hence, IRR lies within 15% and 16%.

$$\text{So, Exact IRR} = r - \frac{\text{PB} - \text{DFr}}{\text{DFrL} - \text{DFrH}}$$

Where :PB = Pay back period;
DFr=Discount factor for interest rate r;
DFrL = Discount factor for lower interest rate and
DFrH = Discount factor for higher interest rate.

$$\begin{aligned} &= 15\% - \frac{5.000 - 5.0188}{5.0188 - 4.8332} \\ &= 15\% - \frac{0.0188}{0.1856} \\ &= 15\% + 0.10\% \\ &= \underline{\underline{15.10\%}} \end{aligned}$$

b) Cash outflows = Tk. 1,000

Cash inflows = Tk. 500

$$\text{PAVF or PB} = \frac{\text{Tk. 1,000}}{\text{Tk. 500}} = \underline{\underline{2.000}}$$

Looking at the Present Value Annuity Table across 3 years row, this factor 2 is found within 23% and 24% columns. Hence, IRR lies within 23% and % and 24%.

$$\begin{aligned} \text{So, Exact IRR} = r &= \frac{\text{PB} - \text{DFr}}{\text{DFrL} - \text{DFrH}} = \frac{2.000 - 2.0114}{2.0114 - 1.9813} = 23\% - \frac{0.0114}{0.0301} \\ &= 23\% - \frac{0.0114}{0.0301} = 23\% + 0.379\% = \underline{23.38\%} \end{aligned}$$

c) Cash outflows = Tk. 1,000

Cash inflows = Tk. 60

$$\text{PAVF or PB} = \frac{1,000}{60} = \underline{16.666}$$

Looking at the Present Value Annuity Table across 55 years (the last year) row, this factor 16.666 is found within 5% and 6% columns. Hence, IRR lies within 5% and 6%.

$$\begin{aligned} \text{So, Exact IRR} = r &= \frac{\text{PB} - \text{DFr } 16.666 - 18.6335}{\text{DFrL} - \text{DFrH}} = \frac{1.9675}{18.6335 - 15.9905} = 5\% - \frac{1.9675}{2.643} = 5\% + 0.74\% \\ &= \underline{5.74\%} \end{aligned}$$

Problem - 2

The Eastern Ltd.'s bonds have 4 years remaining to maturity. Interest is paid annually. The bonds have a Tk. 1,000 par value and the coupon interest rate is 9%.

Compute approximate yield to maturity for the bonds if the current market price is either : (i) Tk. 829 or (ii) Tk. 1,104.

Solution

We know that –

$$\text{YTM} = \frac{\text{INT} + \frac{M - Vd}{N}}{\frac{2(Vd) + M}{3}}$$

In case of (i)

$$\text{YTM} = \frac{90 + \frac{1000 - 829}{4}}{\frac{2(829) + 1000}{3}} = \frac{90 + 42.75}{886} = \frac{132.75}{886} = 0.1498 \quad \text{or} \quad 15\%$$

In case of (ii)

$$\text{YTM} = \frac{90 + \frac{1000 - 1104}{4}}{\frac{2(1104) + 1000}{3}} = \frac{90 - 26}{1069.33} = \frac{64}{1069.33} = 0.059 \quad \text{or} \quad 6\%$$

Problem - 3

What will be the rate of return on a perpetual bond with a Tk. 1,000 par value, at an 8% coupon rate and current market price of : (a) Tk. 600; (b) Tk. 800 and (c) Tk. 1,500 ? Assume interest is paid annually.

Solution

We know that -

$$A^*$$

$r = \frac{A^*}{A_0}$
Where : A^* = Fixed annual interest payment
 A_0 = Market price of the bond.

So, in case of (a) -

$$r = \frac{80}{600} = 0.1333 \text{ or } 13.3\%$$

In case of (b) -

$$r = \frac{80}{800} = 0.10 \text{ or } 10\%$$

In case of (c) -

$$r = \frac{80}{1,500} = 0.053 \text{ or } 5.3\%$$

Stock return is the discount rate which equates the present value of the dividend stream and ending price with the purchase price.

Stock Returns

Stock return is the discount rate, which equates the present value of the dividend stream and ending price with the purchase price. One period stock return (r) may be found out by applying the following formula:

$$r = \frac{\text{Dividends} + (\text{Ending price} - \text{Beginning price})}{\text{Beginning price}}$$

But, if stock holding

period is more than one, then r is found out by applying the following equation for r :

$$P_0 = \sum_{t=1}^N \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

Dividend Discount Models

Dividend discount models are designed to measure the implied stock return under the specific assumptions as to the expected growth pattern of the future dividends.

Perpetual growth model assumes the dividend one period in the future grows at a constant rate in perpetuity.

a) Perpetual Growth Model

Perpetual growth model assumes the dividend one period in the future grows at a constant rate in perpetuity. According to this model the expected return may be measured by using the following equation :

$$r = \frac{D_1}{P_0} + g$$

Again, price of stock (P_0) is found out by using -

$$P_0 = \frac{D_1}{r - g}$$

With the perpetual growth model, we can easily shift from dividend valuation to P/E ratio valuation. In this context, it is assumed that dividend payout ratio, calculated as dividends per share divided by earning per share, would be constant. Therefore,

$$1 - b = \frac{D_1}{E_1}$$

Where : b = Retention ratio.

Problems and Solutions

Problem - 1

You are interested to buy a share of stock of a corporation for Tk. 150. The corporation is expected to pay a Tk. 6 dividend at the end of the year and its market price is expected to be Tk. 165 a share. Calculate your expected return.

Solution

We know that -

$$r = \frac{\text{Dividends} + (\text{Ending Price} - \text{Beginning Price})}{\text{Beginning Price}}$$

$$= \frac{6.00 + (165 - 150)}{150} = \frac{21}{150} = 0.14 \text{ or } \underline{\underline{14\%}}$$

Problem - 2

In the above problem – 1, if the stockholding period is 10 years, instead of 1 year; what is the expected return then?

Solution

We know that -

$$P_0 = \sum_{t=1}^{10} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$
$$150 = \sum_{t=1}^{10} \frac{6}{(1+r)^{10}} + \frac{165}{(1+r)^{10}}$$
$$\text{or, } 150 = \frac{(6+165)}{(1+r)^{10}}$$

$$\text{or, } 150(1+r)^{10} = 171$$

$$\text{or, } (1+r)^{10} = \frac{171}{150} = 1.14$$

$$\text{or, } (1+r)^{10} = 1.14$$

$$\text{or, } r = 1.14 - 1.00$$

$$\text{or, } r = 14\%$$

Problem -3

The Pran Textile Ltd. presently pays a dividend of Tk. 16 per share and the market price per share is Tk. 300. The company expects a dividend growth rate of 12% annually during its lifetime. What is the stock's expected return on investment?

Solution

We know that -

$$r = \frac{D_1}{P_0} + g = \frac{16}{300} + 0.12 = 0.0533 + 0.12 = 0.1733 \quad \text{or, } \underline{17.33\%}$$

An investment risk, also known as financial risk, refers to the chance that an outcome/ return other than the expected one will occur.

Techniques of Measuring Financial Risks

So far we have dealt with only the expected return from holding a security. But, in a world of uncertainty, this return may not be realized fully. Since, this return relates to future; hence, is uncertain. As a result, risk is there in getting actual return. So, risk can be thought of as the possibility that the actual return from holding a security will deviate from the expected return. The greater the extent of such deviation and the

greater the possibility of its occurrence, the greater is said to be the risk of security. Therefore, an investment risk, also known as financial risk, refers to the chance that an outcome/ return other than the expected one will occur.

In measuring financial risk, one is to follow standard deviation method. While following this method probability distribution of the expected return should be considered. Probability distribution refers to a listing of all possible returns, outcomes or events with a probability i.e. a chance of occurrence, assigned to each return. Probabilities can also be assigned to the possible returns from an investment. As for example, you buy a bond, you can expect to receive interest on the bond and those interest payments will provide you with a rate of return on investment. The possible outcomes from this investment are : (i) that the issuer will make the interest payments and (ii) that the issuer will fail to make the interest payments. The higher the probability of default on the interest payments, the riskier the bonds and in turn, the risk, the higher the rate of return you would require to invest in the bond.

Probability distribution refers to a listing of all possible returns, outcomes or events with a probability i.e. a chance of occurrence, assigned to each return.

Standard deviation is a measure of the relative dispersion of a probability distribution. Whenever, we use standard deviation method in measuring financial risk involved in returns from securities; we try to measure the relative dispersion of a probability distribution of the expected returns. This probability distribution can be summarized in terms of two parameters : (i) the expected return and (ii) the standard deviation. The expected return is :

Standard deviation is a measure of the relative dispersion of a probability distribution.

$$R = \sum_{i=1}^n R_i P_i$$

Where, R_i is the return for the i th probability, P_i is the probability of occurrence of that return and n is the total number possibilities. The standard deviation is :

$$\sigma = \sqrt{\sum_{i=1}^n (R_i - \bar{R})^2 P_i}$$

By relating the standard deviation to a specific difference from the expected return, we are able to determine the

probability of occurrence of that return/ outcome. The trade-off between the expected return and risk (standard deviation) is the key factor. Hence, the investors attempt to maximize their expected return, which is a function of the expected return and standard deviation.

The trade-off between expected return and risk (standard deviation) is the key factor.

Problems and Solutions

Problem - 1

Stocks X and Y have the following probability distributions of expected future returns :

Probability	Expected Returns from X	Expected Returns from Y
0.1	-10%	-35%
0.2	2	0
0.4	12	20
0.2	20	25
0.1	38	45

Required :

- Calculate the expected rate of return for X and Y.
- Calculate standard deviation of the expected returns for X and Y.

Solution :

Table showing Calculations of Expected Rate of Return (\bar{R}) and standard Deviation

Prob-ability (P)	R from X (%)	RP _x (%)	R from Y (%)	RP _y	(R _x - \bar{R}_x) ²	(R _y - \bar{R}_y) ²	P(R _x - \bar{R}_x) ²	P(R _y - \bar{R}_y) ²
0.1	-10	-1.00	-35	-3.50	484	2401	48.4	240.1
0.2	2	0.40	0	0	100	196	20.0	39.2
0.4	12	4.80	20	8.00	0	36	0	14.4
0.2	20	4.00	25	5.00	64	121	12.8	24.2
0.1	38	3.80	45	4.50	676	961	67.6	96.1
	$\sum R_x = 62$	$\frac{12.00}{\sum RP_x} = \bar{R}_x$	$\sum R_y = 55$	$\frac{14.00}{\sum RP_y} = \bar{R}_y$			148.8	414.0

$$\sigma R_x = \sqrt{\sum_{i=1}^n (R_i - \bar{R})^2} = \sqrt{148.8} = 12.20\%$$

$$\sigma R_y = \sqrt{414.0} = 20.35\%$$

Problem - 2

Mr. Patwary is thinking investing in security that has the following distribution of possible returns :

Possibility	0.10	0.20	0.30	0.10	0.15	0.15
Possible Return (In %)	- 15	20	15	5	25	28

Required :

What is the expected rate of return and standard deviation of the expected returns ?

Solution :

Table showing calculations of R_i and \bar{R}

Possibility	Returns (R_i) (%)	R_p	$(R_i - \bar{R})$	$(R_i - \bar{R})^2$	$P(R_i - \bar{R})^2$
0.10	- 15	- 1.50	- 30.45	927.20	92.72
0.20	20	4.00	4.55	20.70	4.14
0.30	15	40.50	- 0.45	0.20	0.06
0.10	5	0.50	- 10.45	109.20	10.92
0.15	25	3.75	9.55	91.20	13.68
0.15	28	4.20	12.55	157.50	23.65
		$15.45 = \bar{R}$		1306.00	$145.17 = \sum (R_i - \bar{R})^2$

$$\sigma = \sqrt{\sum_{i=1}^n (R_i - \bar{R})^2 P_i} = \sqrt{145.17} = 12.05\%$$

Review Questions

A. Short Questions

1. What is IRR ? How can it be calculated ?
2. What is bond return ? How it can be measured ?
3. How are returns from perpetual bonds measured ? Explain.
4. What is stock return ? How it can be measured if the stock is held for one year only ?
5. What does financial risk mean ? Give some examples.
6. What is probability ? How it is related to risk and return ?

Broad Questions

1. How can returns from maturity bonds be measured with (i) Zero coupon bonds and (ii) Coupon bonds ?
2. Examine the dividend discount model. How can dividend valuation be shifted to P/E ratio valuation ?
3. How expected return and standard deviation of such return are measured ? Illustrate your answer.

Review Problems

Problem - 1

Securities X and Y have the following characteristics

Security X		Security Y	
Return	Probability	Return	Probability
30%	0.10	-20%	0.05
20%	0.20	10%	0.25
10%	0.40	20%	0.30
5%	0.20	30%	0.30
-10%	0.10	40%	0.10

You are required to calculate (a) The expected return of the securities, (b) Standard deviation of return for each security.

Problem - 2

The East and West Companies have the following probability distribution of returns :

Economic conditions	Probability	Returns (%)	
		East	West
High growth	0.1	32	30
Normal growth	0.2	20	17
Slow growth	0.4	14	6
Stagnation	0.2	-5	-12
Decline	0.1	-10	-16

You are required (a) to determine the expected return of the companies, (b) standard deviation of return for each company.

Problem - 3

It is now January 1, 2000, and you are considering the purchase of an outstanding Puckett Corporation bond that was issued on January – 1, 1998. The Puckett bond has a 9.5 percent annual coupon and a 30 – year original maturity (it matures on December 31, 2027). Interest rate have declined since the bond was issued, and the bond now is selling at 116.575 percent of par, or \$1,165.75. You want to determine the yield to maturity for this bond.

- a) Approximate the yield to maturity for the Puckett bond in 2000.
- b) What is the actual yield to maturity in 2000 for the Puckett bond?

Problem - 4

Case Study :

Assume that you recently graduated with a major in finance, and you just landed a job in the trust department of large regional bank. Your first assignment is to invest \$1,00,000 from an estate for which the bank is trustee. Because the estate is expected to be distributed to the heirs in about one year, you have been instructed to plan for a one – year holding period. Further, your boss has required you to the following investment alternatives, shown with their probabilities and associated outcomes. (Disregard for now the items at the bottom of the data; you will fill in the blanks later)

State of the Economy	Returns on Alternative Investment						
	Estimated Rate of Return						
	Prob.	T -Bills	High Tech	Collections	U.S. Rubber	Market Portfolio	2 – Stock Portfolio
Recession	0.1	8.0%	-22.0%	28.0%	10.0%	-13.0%	
Below Average	0.2	8.0	-2.0	14.7	-10.0	1.0	
Average	0.4	8.0	20.0	0.0	7.0	15.0	
Above Average	0.2	8.0	35.0	-10.0	45.0	29.0	
Boom	0.1	8.0	50.0	-20.0	30.0	43.0	
\hat{R}							
σ							

The bank’s economic forecasting staff has developed probability estimates for the state of the economy, and the trust department has a

sophisticated computer program that was used to estimate the rate of return on each alternative under each state of the economy. High Tech Inc. is an electronics firm; Collections Inc. collects past due debts; and U.S. Rubber manufactures tires and various other rubber and plastics products. The bank also maintains an “index fund”, what owns a market – weighted fraction of all publicly traded stocks; you can invest in that fund and thus obtain average stock market results. Given the situation as described, answer the following questions:

a) Calculate the expected rate of return on each alternative and fill in the row for \bar{R} in the preceding table.

b) You should recognize that basing a decision solely on expected return is only appropriate for risk-neutral individuals. Because the beneficiaries of the trust, like virtually everyone, are risk averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns.

i) Calculate this value for each alternative, and fill in the row for σ in the preceding table.

2) What type of risk is measured by the standard deviation?

3) Suppose you created a two stock portfolio by investing \$ 50,000 in Hi Tech and \$ 50,000 in Collections :

(i) Calculate the expected return, the standard deviation and the coefficient of variation for this portfolio.

(ii) How does the riskiness of this two stock portfolio compared to that of the individual stocks if they were held in isolation ?

Lesson-7: Market Risk and Return (MRR)

After successful completion of this lesson 7, you should be able –

- To have an idea on the concepts of market risk and return and market efficiency;
- To understand the techniques of portfolio security analysis and selection;

Return means outcome of an investment.

Concepts of Market Risk and Return Market Efficiency

a) Concept of Return: Simply stated, return means outcome of an investment. If an investor invests money in real assets or financial assets; then he may get return from real assets or from financial assets. Again return may be actual or expected. Return is the reward from undertaking the investment. Return on a typical investment consists of two components viz., yield and capital gain. Yield is the income component of a security's returns. Capital gain is the change in price on a security over some period of time.

Risk is the chance of occurrence of the deviation between the expected and actual event.

b) Concept of Risk and Market Risk : Risk can be defined as the chance that some event other than expected will occur. That is, risk is the chance of occurrence of the deviation between the expected and actual event. The risk associated with the investment in securities is known as investment risk. The investment risk, is, thus related to the possibility of actually earning a return other than expected – the greater the variability of the possible outcomes, the riskier the investment. The portion of a security's risk that can be eliminated is called diversifiable or firm-specific or unsystematic risk; whereas, that portion of a security's risk that cannot be eliminated is called non-diversifiable or market or systematic risk. Market risks are associated with economic or market factors that systematically affect most firms. As for example, war, inflation, recessions, depressions and high interest rates are known as the economic or market factors, which affect securities of most of the firms. Because most securities tend to be affected similarly (negatively) by these market conditions, systematic risk cannot be eliminated by portfolio diversification.

The investment risk, is, thus related to the possibility of actually earning a return other than expected – the greater the variability of the possible outcomes, the riskier the investment.

Market risks are associated with economic or market factors that systematically affect most firms.

c) Concept of Market Efficiency: Market efficiency means that the market prices of the securities represent the security market's consensus estimate of the value of those securities. If the market is efficient, it uses all information available to it in setting security price. Investors who choose to hold a security are doing so because their information lead them to think that the security is worth at least its current market price. And, the persons who do not buy the security interpret their information as a lower appraisal.

Market efficiency means that the market prices of the securities represent the security market's consensus estimate of the value of those securities.

An efficient market (EM) is defined as one in which the prices of all securities quickly and fully reflect all available information about the assets. This concept postulated that investors will assimilate all relevant information into prices in making their buy and sell decisions. Therefore, the current price of a stock reflects:

An efficient market (EM) is defined as one in which the prices of all securities quickly and fully reflect all available information.

An efficient capital market exists when the security market prices reflect all available public information about the economy, financial/ capital markets and the specific firm involved.

1. All known information, including :
 - Past information (e.g., last year's or last quarter's earnings)
 - Current information as well as events that have been announced but are still forthcoming (such as a stock split)
2. Information that can reasonably be inferred; for example, if many investors believe that interest rates will decline soon, prices will reflect this belief before that actual decline occurs.

An efficient capital market exists when the security market prices reflect all available public information about the economy, financial/ capital markets and the specific firm involved. The implication is that the market price of the individual security adjusts very rapidly to new market information. As a result, security prices are said to fluctuate randomly about their 'intrinsic' values. Relatively to the degree of market information, market efficiency may be of some forms or stages which are examined in the next sub-sections.

Stages of Market Efficiency

Three stages of market efficiency are there namely weak form of efficiency, semi strong form of efficiency and strong form of efficiency.

Three stages of market efficiency are there namely weak form of efficiency, semi strong form of efficiency and strong form of efficiency.

In the weak form of efficiency, security prices reflect the market information contained in the record of past prices. In this stage of market efficiency, it is impossible to make consistently superior profits by studying past prices. Price will follow a random walk.

In the semi strong form of efficiency, it is required that the security prices reflect not just the past prices but all other published information. In this case, prices will adjust immediately to the public information.

Finally, in the strong form of efficiency, security prices reflect all the available information, past, present, published and unpublished. Such information can be acquired by painstaking analysis of the company and the economy. In such a market we would observe lucky and unlucky investors.

The hypothesis that the security markets are efficient will be true only if a sufficiently large number of investors are not in doubt about its efficiency and behave accordingly. In other words, the theory requires that there be sufficiently a large number of market participants who, in their attempts, for earnings profits, promptly receive and analyze all the information that are publicly available relevant to firms whose securities they follow.

Conditions of Market Efficiency

It can be shown that an efficient market can exist if the following events occur:

1. A large number of rational profit maximizing investors exist who actively participate in the market by analyzing, valuing and trading stocks. These investors are price takers; that is, one participate alone cannot affect the price of a security.

2. Information is costless and widely available to market participants at approximately the same time.
 3. Information is generated in a random fashion such that announcements are basically independent of one another.
 4. Investors react quickly and fully to the new information, causing stock prices to adjust accordingly.
- These conditions may seem strict, and in some sense they are. Nevertheless, consider how closely they parallel the actual investments environment.

Concept of Portfolio Security: Its Analysis and Selection or Measurement

By portfolio security we mean the combination of some securities, whether stocks or bonds. That is, instead of investing in one type of security, whenever an investor invests in more than one type security; then that is known as portfolio security. Now a days, since most of the rational investors hold portfolios of securities, and hence, they are more concerned with the analysis and selection of portfolio securities. Therefore, portfolio security analysis refers to the analysis of portfolio risk and portfolio returns for selecting the portfolio of the existing securities available in the markets.

Portfolio security analysis refers to the analysis of portfolio risk and portfolio returns for selecting the portfolio of the existing securities available in the markets.

When we analyze investment returns and risks, we must be concerned with the total portfolio held by an investor. Individual security risks and returns are important; but it is the return and risk to the investor's total portfolio that ultimately matters because investment opportunities can be enhanced by packaging them together to form portfolio.

Portfolio Expected Return and Its Measurement

The expected return for portfolio security is a weighted average of expected returns for securities making up that portfolio. So, the expected return on the portfolio security is easily measured as the weighted average of the individual securities' expected returns. The percentages of a portfolio's total value that are invested in each portfolio asset are referred to as **portfolio weights**, which will denote by W . The combined portfolio weights are assumed to sum to 100 percent of total investible funds or 1.0 indicating that all the portfolio funds are invested.

The expected return for portfolio security is a weighted average of expected returns for securities making up that portfolio.

The expected return on any portfolio p can be calculated as

$$E(R_p) = \sum_{i=1}^n w_i E(R_i)$$

- Where, $E(R_p)$ = the expected return on the portfolio
 w_i = the portfolio weight for the i th security
 $\sum w_i$ = 1.0
 $E(R_i)$ = the expected return on the i th security
 n = the number of different securities in the portfolio.

Portfolio Risk and Its Measurement

Portfolio risk is total risk of all securities forming the portfolio.

Risk associated with the portfolio security is called portfolio risk. That is, portfolio risk is total risk of all securities forming the portfolio. Logically, the risk of an investment should not be evaluated in isolation; rather the risk associated with all the securities forming the investment portfolio should be evaluated.

Risk is measured by the variance (or standard deviation) of the portfolio's return, exactly as in the case of each individual security.

The portfolio risk is not a simple weighted average of the standard deviations of the individual securities. Portfolio risk depends not only on the riskiness of the securities constituting the portfolio but also on the relationships among those securities. Risk is measured by the variance (or standard deviation) of the portfolio's return, exactly as in the case of each individual security.

It is at this point that the basis of modern portfolio theory emerges, which can be stated as follows: Although the expected return of a portfolio is a weighted average of its expected returns, portfolio risk (as measured by the variance or standard deviation) is not a weighted average of the risk of the individual securities in the portfolio. Symbolically,

$$E (R_p) = \sum_{i=1}^n w_i E (R_i) \dots \dots \dots (1)$$

But

$$\sigma_p^2 \neq \sum_{i=1}^n w_i \sigma_i^2 \dots \dots \dots (2)$$

Precisely because the equation no. 2 is an inequality, investors can reduce the risk of a portfolio beyond what it would be if risk were, in fact, simply a weighted average of the individual securities' risk.

The risk of a well diversified portfolio depends on the market risk of the securities included in the portfolio.

(i) Determining Two-Security Case

The risk of a portfolio, as measured by the standard deviation of returns, for the case of two securities, 1 and 2, is

$$\sigma_p = \{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2(w_1)(w_2)(\rho_{1,2})\sigma_1 \sigma_2\}^{1/2}$$

Now, the question arises : How does individual security affect portfolio risk ?

The risk of a well diversified portfolio depends on the market risk of the securities included in the portfolio. The risk of an individual security of a well diversified portfolio is not same of that portfolio.

We have to first determine market risk and then determine how sensitive it is with the market portfolio. The sensitivity is (β).

- The portfolio of the stocks has a market β = 1.
- β > 1 tends to amplify the overall movement of the market.
- β between 0 to 1 indicates movement of stock in same direction but not as the same rate (i.e. slow move).

- Market risks accounts for most of the risk of a well diversified portfolio.
- β of an individual security measures its sensitivity to market movement.

Covariance of securities i's return and market return

$$\beta_i = \frac{\text{Covariance of securities i's return and market return}}{\text{Variance of the market return}}$$

$$= \frac{\sigma_{im}}{\sigma_{im}^2}$$

When the portfolio contains more than two securities, then multiple security portfolio occurs.

Multiple Security Portfolio Analysis and Measurement

When the portfolio contains more than two securities, then multiple security portfolio occurs. The same principles hold when we go to multiple security portfolio.

Portfolio risk is a function of each individual security's risk and the covariance between the returns on the individual securities.

Multiple Security Portfolio / Risk can be measured as follows :

The two – security case can be generalized to the n – security case. Portfolio risk can be reduced by combining assets with less than perfect positive correlation. Furthermore, the smaller the positive correlation, the better.

Portfolio risk is a function of each individual security's risk and the covariance between the returns on the individual securities. Stated in terms of variance, portfolio risk is

$$\sigma_i^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

Where,

- σ_p^2 = the variance of the return on the portfolio
- σ_i^2 = the variance of return for security i
- σ_{ij} = the variance between the returns for securities I and j

w_i = the portfolio weights or percentage of investable funds invested in security i

$\sum_{i=1}^n \sum_{j=1}^n$ = a double summation sign indicating that n^2 numbers are to be added together $i=1 j=1$ (i.e., all possible pairs of values for i and j)

Investors can identify efficient portfolios by specifying an expected portfolio return and minimizing the portfolio risk at this level of return.

Efficient Portfolio

Markwitz's approach to portfolio selection is that an investor should evaluate portfolios on the basis of their expected returns and risk as measured by the standard deviation. He was first to derive the concept of

an efficient portfolio, defined as one that has the smallest portfolio risk for a given level of expected return or the largest expected return for given level of risk. Investors can identify efficient portfolios by specifying an expected portfolio return and minimizing the portfolio risk at this level of return. Alternatively, they can specify a portfolio for this level of risk. Rational investors will seek efficient portfolios because these portfolios are optimized on the two dimensions of most importance to investors, expected return and risk.

Review Questions

Short Questions

1. What is an efficient market ? Under what circumstances an efficient market can exist?
2. What is a portfolio security? Give examples.
3. How many and which factors determine portfolio risk?
4. How does individual security affect portfolio risk? Explain.

Broad Questions

5. How would you define market risk and return? What is market efficiency? Explain the stages of market efficiency.
6. Discuss the various aspects of portfolio security analysis.

Review Problems

Problem – 1

Stocks A and B have the following returns during 1998-2002:

Year	Stock A's Returns (In %)	Stock B's Returns(In %)
1998	17	-5.50
1999	36	32.10
2000	23	29.40
2001	-18	16.60
2002	29	23.50

Calculate the portfolio return and risk of the securities if 40% weights are placed in Stock A and the remaining 60% weight are placed in Stock B.

Problem - 2

A portfolio consists of three securities X, Y and Z with the following returns :

Particulars	X	Y	Z	r
Expected Returns (%)	19	26	32	
Standard Deviation (%)	22	31	35	
Correlation Co-efficient				
XY				0.4
XZ				0.7
YZ				-0.3

If the above securities are equally weighted, how much is the return and risk of the portfolio of these three securities ?

Lesson–8: Capital Asset Pricing Model (CAPM) and Expected Return and Risk (ERR)

After attentively studying this lesson 8, you should be able -

- To grasp the concept of Capital Asset Pricing Model (CAPM);
- To identify the assumptions of CAPM;
- To understand the concept of Beta and Beta Coefficient;
- To know portfolio Beta Coefficient and
- To know how to determine expected return and risk (ERR)

Concept of Capital Asset Pricing Model (CAPM)

A model used to determine the required return on asset, which is based on the proposition that any asset's return should be equal to the risk-free rate of return plus a risk premium that reflects the asset's non-diversifiable risk.

Assumptions of CAPM

CAPM is based on a number of assumptions. The most important assumptions are :

- (1) Market efficiency : The capital markets are efficient. The capital market efficiency implies that share prices reflect all available information.
- (2) Risk aversion : Investors are risk averse. They evaluate a security's return and risk in terms of the expected return and variance or standard deviation respectively. They prefer the highest expected returns for a given level of risk.
- (3) Homogeneous expectations : All investors have the same expectations about the expected return and risk of securities.
- (4) Single time period : All investor's decisions are based on single time period.
- (5) Risk-free rate : All investors can lend or borrow at a risk-free rate of interest.
- (6) Individuals seek to minimize the expected utility of their portfolios over a single period planning horizon.
- (7) The market is perfect : There are no taxes. There are no transaction costs. Securities are completely divisible, the market is competitive.
- (8) The quantity of risky securities in the market is given.

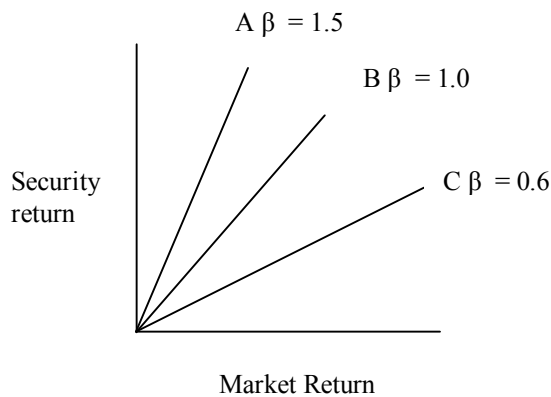
The logic of CAPM can be extended to price individual securities and determine the required rate of return from individual securities.

Concept of Beta and Beta Co-efficient

Beta is a measure of the market risk or systematic risk of a security that cannot be avoided through diversification. Beta is a relative measure of risk of an individual stock in relation to the market portfolio of all stocks. Whereas, the measure of a security's sensitivity to market fluctuations is called Beta Coefficient and is generally designated with the Greek symbol for β (beta). Beta is the key element of the CAPM. Therefore,

Beta is a relative measure of risk of an individual stock in relation to the market portfolio of all stocks. Whereas, the measure of a security's sensitivity to market fluctuations is called Beta Coefficient.

Beta Co-efficient (β) is a measure of the extent to which the return on a given security moves with the capital market. The following Figure presents betas of 1.5(A), 1.0 (B) and 0.6 (C).



If the security's returns move (less) than the market's returns as the latter changes, the security's return have more (less) volatility (fluctuations in price) than those of the market. For example, a security whose returns rise or fall on average 15 percent when the market returns rise or fall 10 percent is said to be an aggressive, or volatile security.

Securities with different slopes have different sensitivities to the returns of the market index.

Securities with different slopes have different sensitivities to the returns of the market index. If the slope of this relationship for a particular security is a 45 degree angle, as shown for security B in the above figure, the beta is 1.0. This means that for every 1 percent change in the market's return, on average this security's returns change 1 percent. The market portfolio has a beta of 1.0.

The aggregate market has a beta of 1.0. More volatile (risky) stocks have betas larger than 1.0, and less volatile (risky) stocks have betas smaller than 1.0. As a relative measure of risk, beta is very convenient.

In summary, the aggregate market has a beta of 1.0. More volatile (risky) stocks have betas larger than 1.0, and less volatile (risky) stocks have betas smaller than 1.0. As a relative measure of risk, beta is very convenient. Beta is useful for comparing the relative systematic risk of different stocks and, in practice, is used by investors to judge a stock's riskiness. Stocks can be ranked by their betas. Because the variance of the market is a constant across all securities for a particular period, ranking stocks by beta is the same as ranking them by their absolute systematic risk. Stock with high (low) betas are said to be high (low) risk securities.

Using portfolio beta co-efficient, the expected return for an individual security of a portfolio can be measured.

Portfolio Beta Co-efficient

Using portfolio beta co-efficient, the expected return for an individual security of a portfolio can be measured. For the individual security, then, the relevant risk is not the standard deviation of the security itself (total risk), but the marginal effect the security has on the standard deviation of an efficiently diversified portfolio (systematic risk). As a result, a security's expected return should be related to its degree of systematic risk, not to its degree of total risk. Systematic risk is the thing that matters to an investors holding a well diversified portfolio. If we assume that unsystematic risk is diversified away, the expected rate of return for stock j is

$$\bar{R}_j = R_f + (\bar{R}_m + R_f)\beta_j$$

where again R_f is the risk-free rate, \bar{R}_m is the expected overall return for the market portfolio, and β_j is the beta coefficient for security j as defined earlier. The greater the beta of a security, the greater the risk and the greater the expected return required. By the same token, the lower the beta, the lower the risk, the more valuable it becomes, and the lower the expected return required.

Determination of Expected Return and Risk (ERR)

In earlier section, the formula for determining portfolio expected return of an individual security is explained. In earlier section, the formula for determining portfolio risk of an individual security is explained. In that section, formula for calculating portfolio risk of the two-security case and portfolio risk of the multiple security case are explained. The following problems relate to these returns and risks.

Problems and Solutions

Problem – 1

Consider a three stock portfolio consisting of stocks X, Y, and Z with expected returns of 12%, 20% and 17% respectively. Assume that 50% of investible funds is invested in stock X, 30% in Y and 20% in Z.

Calculate portfolio expected return.

Solution

We know that –

$$\begin{aligned} E(R_p) &= \sum_{i=1}^n W_i E(R_i) \\ &= 0.5 (12\%) + 0.3 (20\%) + 0.2 (17\%) \\ &= 6\% + 6\% = 12\% \\ &= \underline{12\%} \end{aligned}$$

Problem - 2

Four securities have the following expected returns:

A = 15%, B = 12%, C = 30% and D = 22%

Calculate the expected returns for a portfolio consisting of all four securities under the following conditions:

- The portfolio weights are 25% each;
- The portfolio weights are 10% in A with remainder equally divided among the other three securities and
- The portfolio weights are 25% in A, 28% in B, 22% in C and 25% in D.

Solution

$$\begin{aligned} \text{(a) } E(R_p) &= .25 (15\%) + .25 (12\%) + .25 (30\%) + .25 (22\%) \\ &= 3.75\% + 3.00\% + 7.50\% + 5.50\% \\ &= \underline{19.75\%} \\ \text{(b) } E(R_p) &= .10 (15\%) + .30 (12\%) + .30 (30\%) + .30 (22\%) \\ &= 1.50\% + 3.60\% + 9.00\% + 6.60\% \\ &= \underline{20.70\%} \\ \text{(c) } E(R_p) &= .25 (15\%) + .28 (12\%) + .22 (30\%) + .25 (22\%) \\ &= 3.75\% + 3.36\% + 6.60 + 5.50\% \\ &= \underline{19.21\%} \end{aligned}$$

Problem - 3

Stocks X and Y have the following historical returns :

Year	Stock X's Returns	Stock Y's Returns
1996	-19%	-14.50%
1996	33%	21.80%
1998	15%	30.50%
1999	0.50%	-7.60%
2000	27%	26.30%

Calculate the portfolio risk of the securities of 50% weights are placed in each stock A and B.

Solution

We know that in case of the two-security case,

$$\sigma_p = [W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2(W_1)(W_2)(P_{12})\sigma_1\sigma_2]^{1/2}$$

Therefore, first of all we are to find out σ of the individual security. The formula for calculating σ_R is explained in the previous Lesson – 4 which goes as follows :

$$\sigma_R = \sqrt{\sum_{i=1}^n (R_{ix} - \bar{R}_x)^2 p_i}$$

Table showing calculations of Standard Deviation (σ)₂

Year	Return (R _{ix})	(R _{ix} - \bar{R}_x)	(R _{ix} - \bar{R}_x) ²	Return Y (R _{iy})	(R _{iy} - \bar{R}_y)	(R _{iy} - \bar{R}_y) ²
1996	-18.00	-29.5	870.25	-14.50	-25.80	665.64
1997	33.00	21.5	462.25	21.80	10.50	110.25
1998	15.00	3.50	12.25	30.50	19.20	368.64
1999	0.50	-11.00	121.00	-7.60	-18.90	357.21
2000	27.00	15.50	24.25	26.30	15.00	256.00
Total	$\bar{R}_x =$ 11.50		1706	$\bar{R}_y =$ 11.30		1757.74 $= (R_{iy} - \bar{R}_y)^2$

$$\sigma_{Rx} = \sqrt{1706 \times 0.50} = 29.21$$

$$\sigma_{Ry} = \sqrt{1757.74 \times 0.50} = 29.65$$

$$\begin{aligned} \sigma_p &= [(0.5)^2 \times (29.21)^2 + (0.50)^2 \times (29.65)^2 + 2(0.50)(0.50)(29.21)(29.65)]^{1/2} \\ &= [213.25 + 219.78 + 433.04]^{1/2} \\ &= \underline{29.43\%} \end{aligned}$$

Problem - 4

A portfolio consists of three securities P, Q and R with the following information :

Particulars	P	Q	R	Correlation Co-efficient
Expected Return (%)	27	23	21	
Standard Deviation (%)	31	27	25	
Correlation Co-efficient				
PQ				-0.5
QR				+0.6
PR				0.7

If the securities are equally weighted, how much is the risk and return of the portfolio of these three securities.

Solution

The portfolio return is --

$$E(R_p) = (27) (1/3) + (23) (1/3) + (21) (1/3)$$

$$= 9 + 7.67 + 7 = \underline{23.67\%}$$

The portfolio risk is =

$$\sigma_p = [(31)^2 (1/3)^2 + (27)^2 (1/3)^2 + (25)^2 (1/3)^2 + 2.(1/3) (1/3) (-0.5) (27) (31) + 2(1/3) (1/3) (0.6)(27) (25) + 2 (1/3) (1/3) (0.7)(31)(25)]^{1/2}$$

$$= [106.67 + 80.92 + 69.38 - 92.00 + 89.91 + 120.54]^{1/2}$$

$$= (375.54)^{1/2}$$

$$\therefore \sigma_p = \sqrt{375.54} = \underline{19.38\%}$$

Problem – 5

Corliss Services Inc., provides maintenance services to commercial buildings. Presently, the beta on its stock is 1.08. The risk – free rate is now 10%, the expected return for the market portfolio is 15%. Corliss is expected to pay a \$2 per share dividend at the end of the year and to grow in nominal terms at a rate of 11% per annum for many years to come. Based on the CAPM and other assumptions, what is the market price per share of the stock?

Solution

We know that according to CAPM, Return on Stock is --

$$\bar{R}_j = R_j + (\bar{R}_m - R_j) \hat{\beta}_j$$

Where : \bar{R}_j = Return on Stock

R_j = Risk free rate

\bar{R}_m = Expected overall return for the market portfolio

$\hat{\beta}_j$ = Beta Co-efficient for security.

$$\begin{aligned}\therefore \bar{R}_j &= 10\% + (15\% - 10\%) 1.08 \\ &= 10\% + 5.41\% = \underline{15.41\%}\end{aligned}$$

We use the perpetual dividend growth model, we would have :

$$P_0 = \frac{D_1}{R - g}$$

$$P_0 = \frac{.110}{.1541 - .110} = \frac{.110}{0.0441} = \$45.35$$

Where : P_0 = Market price per share;
 D_1 = Expected dividend at the end of t period;
 R = Return on Stock and
 g = Dividend growth rate.

Review Questions

Short Questions

1. What is CAPM ? What are its assumptions ?
2. What are Beta and Beta Coefficient ?
3. What is Portfolio Beta Coefficient ?

Broad Questions

4. What is portfolio expected return and risk ? How they can be measured ? Explain.
5. What is the concept of Beta and Beta Coefficient ? Explain Beta Coefficient graphically.
6. According to CAPM, how expected return for an individual security can be measured ? Give example.
7. How are expected returns and risks are determined in cases of : (a) One security; (b) Two security and (c) Multiple security ?

Review Problems

Problem - 1

Calculate the expected return and risk for General Foods for 2003, given the following information:

Probabilities : 0.15 0.20 0.40 0.10 0.15

Possible Returns (%) : 20 16 12 5 -5

Problem – 2

Securities X and Y have the following Characteristics :

Security X		Security Y	
Return (%)	Probability	Return (%)	Probability
30	0.10	-20	0.05
20	0.20	10	0.25
10	0.40	20	0.30
5	0.20	30	0.30
-10	0.10	40	0.10

Required :

- (a) Calculate the expected return and risk of return for each security.
- (b) Calculate the expected return and risk of the return for the portfolio of X and Y, placing equal weight to each.

Problem – 3

Securities D, E and F have the following characteristics with respect to expected return, standard deviation, and the correlation between them:

Company	Correlation Coefficients				
	R	SD	D – E	D – F	E – F
D	.08	.02	.4	.6	
E	.15	.16	.4		.8
F	.12	.08		.6	.8

What is the expected return and standard deviation of a portfolio composed of equal investments in each ?

Case Study

Assume that you recently graduated with a major in finance, and you just landed a job in the trust department of a large regional bank. Your first assignment is to invest \$1,00,000 from an estate for which the bank is trustee. Because the estate is expected to be distributed to the heirs in about one year, you have been instructed to plan for a one-year holding period. Further, your boss has required you to the following investment alternatives, shown with their probabilities and associated outcomes. (Disregard for now the items at the bottom of the data; you will fill in the blanks later.)

State of the Economy							
	Prob.	T-Bills	High Tech	Collections	U.S.Rubber	Market Portfolio	2-Stock Portfolio
Recession	0.1	8.0%	-22.0%	28.0%	10.0%	-13.0%	
Below Average	0.2	8.0	-2.0	14.7	-10.0	1.0	
Average	0.4	8.0	20.0	0.0	7.0	15.0	
Above Average	0.2	8.0	35.0	-10.0	45.0	29.0	
Boom	0.1	8.0	50.0	-20.0	30.0	43.0	
\hat{R}							
σ							

The bank’s economic forecasting staff has developed probability estimates for the state of the economy, and the trust department has a sophisticated computer program that was used to estimate the rate of return on each alternative under each state of economy. High Tech Inc. is an electronics firm; Collections Inc. collects past due debts; and U.S. Rubber manufactures tires and various other rubber and plastic products. The bank also maintains an “index fund”, what owns a market –

weighted fraction of all publicly traded stocks; you can invest in that fund and thus obtain average stock market results. Given the situation as described, answer the following questions :

- (a) Calculate the expected rate of return on each alternative, and fill in the row for σ in the preceding table.
- (b) What type of risk is measured by the standard deviation ?
- (c) Suppose you created a two-stock portfolio by investing \$50,000 in High Tech and \$50,000 in Collections.
 - (i) Calculate the expected return (k_p), the standard deviation (σ_p) and the coefficient of variation (CV_p) for this portfolio and fill in the appropriate rows in the preceding table.
 - (ii) How does the riskiness of this two-stock portfolio compare to the riskiness of the individual stocks if they were held in isolation.