Logarithm

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The purpose of this unit is to equip the learners with the concept of logarithm. Under the logarithm, the topics covered are nature of logarithm, laws of logarithm, change the base of logarithm, anti-logarithm and its operation followed by ample examples.

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Lesson-1: Nature and Basic Laws of Logarithm

After studying this lesson, you should be able to:

- Discuss the nature of logarithm;
- Identify the basic laws of operation of logarithm;
- Explain the characteristics and mantissa of logarithm.

Meaning of a Logarithm

Logarithm is the important tool of modern mathematics. If $a^x = n$, then x is said to be the logarithm of the number 'n' to the base 'a'. Symbolically it can be expressed as follows: $log_a\, n = x$. In this case $a^x = n$ is an exponential form and $log_a\, n = x$ is a logarithmic form. The object of logarithm is to make common calculations less laborious and the method consists in replacing multiplication by addition and division by subtraction.

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Logarithm to the base 'e' is called *'natural logarithm'* and when the base is 10, the logarithm is called *'common logarithm'*. For example,

(i) $5^3 = 125 \rightarrow \log_5 125 = 3$, i.e. the logarithm of 125 to the base 5 is equal to 3.

(ii) $(64)^{\frac{1}{6}} = 2 \rightarrow \log_2 64 = \frac{1}{6}$, i.e. the logarithm of 64 to the base 2 is equal to $\frac{1}{6}$.

Similarly,

Exponential form		Logarithmic form
$2^3 = 8$	$\rightarrow$	$log_2 8 = 3$
$10^2 = 100$	$\rightarrow$	$log_{10} 100 = 2$
$2^{-2} = \frac{1}{4}$	$\rightarrow$	$log_2 \frac{1}{4} = -2$
$3^0 = 1$	$\rightarrow$	$log_3 1 = 0$

or

Logarithmic form		Exponential form
$log_4 64 = 3$	$\rightarrow$	$4^3 = 64$
$log_p R = Q$	$\rightarrow$	$P^Q = R$
$log_{10} 10 = 1$	$\rightarrow$	$10^1 = 10$
$log_5 1 = 0$	$\rightarrow$	$5^0 = 1$

Fundamental Properties and Laws of Logarithms

The fundamental properties and laws of logarithm are as follows:

(1) The logarithm of the production of two factors is equal to the sum of their logarithms; i.e., $log_a mn = log_a m + log_a n$.

(2) The logarithm of quotient is equal to logarithm of the numerator minus the logarithm of the denominator; i.e., $log_a \left(\frac{m}{n}\right) = log_a m - log_a n$.

(3) The logarithm of any power of a number is equal to the product of the index of the power and the logarithm of the number; i.e. $log_a m^x = x \, log_a m$.

(4) Base changing formula: The formula which tells us how to change from one base to another is : $log_b n = \frac{\log_a n}{\log_a b}$

i.e., $(log_b n)(log_a b) = log_a n$

Characteristics and Mantissa of a Logarithm

The logarithm of a number consists of two parts. The first part is called characteristics and the second part are termed as mantissa

The logarithm of a number consists of two parts: (i) an integer positive, negative or zero (ii) a positive or negative proper fraction. The first part is called characteristics and the second part are termed as mantissa.

Since $10^0 = 1$ $\quad \therefore log\ 1 = 0$
$10^1 = 10$ $\quad \therefore log\ 10 = 1$
$10^2 = 100$ $\quad \therefore log\ 100 = 2$
$10^3 = 1000$ $\quad \therefore log\ 1000 = 3$
$10^4 = 10,000$ $\quad \therefore log\ 10,000 = 4$

Similarly, since $10^{-1} = \frac{1}{10} = 0.1,$ $\quad \therefore log\ 0.1 = -1$

$10^{-2} = \frac{1}{100} = 0.01,$ $\quad \therefore log\ 0.01 = -2$

$10^{-3} = \frac{1}{1000} = 0.001,$ $\quad \therefore log\ 0.001 = -3$

$10^{-4} = \frac{1}{10000} = 0.0001,$ $\quad \therefore log\ 0.0001 = -4.$

In general, the logarithm of a number containing *n* digits only in its integral part is $\{(n - 1) + a\}$ fraction and the logarithm of a number having N zeros just after the decimal point is $\{-(n+1) + a\}$ fraction.

Let us take some examples on logarithm.

Example-1:

If $log_x 625 = 4$; find the value of x.

Solution:

$log_x\ 625 = 4$ can be expressed in exponential form as

$$x^4 = 625$$
$$or,\ x^4 = 5^4$$
$$or,\ x = 5^{\frac{4}{4}} = 5$$

Example-2:

If $\log_{\sqrt{27}} x = -\frac{4}{3}$, find the value of x.

Solution:

Expressing $\log_{\sqrt{27}} x = -\frac{4}{3}$ in the exponential form, we get $\left(\sqrt{27}\right)^{-\frac{4}{3}} = x$

$$\text{or,}\quad x = \left(\sqrt{3^3}\right)^{-\frac{4}{3}}$$

$$\text{or,}\quad x = \left(3^{\frac{3}{2}}\right)^{-\frac{4}{3}}$$

$$\text{or,}\quad x = 3^{-2} = \frac{1}{3^2}$$

$$\text{or,}\quad x = \frac{1}{9}$$

Example-3:

If $10^x = 8$, find the value of x.

Solution:

Here $10^x = 8$ can be expressed in logarithmic form as, $log_{10}\ 8 = x$

Therefore, $x = log_{10}\ 8 = 0.9030$ (by using scientific calculator).

Example-4:

The logarithm of a number is –3.153. Find the characteristics and mantissa.

Solution:

Let $log\ N = -3.153$

$= (-3 - 0.153) = (-3 - 1 + 1 - 0.153) = -4 + 0.847$

$\therefore$ The characteristics is – 4 and mantissa is 0.847.

Example-5:

Find the logarithm whose logarithm is 2.4678.

Solution:

From the Anti-log Table,

For mantissa 0.467, the number = 2931

For mean difference 8, the number = 5

$\therefore$ For mantissa 0.4678, the number = (2931 + 5) = 2936.

The characteristics is 2, therefore the number must have 3 digits in the integral part.

Hence, antilog 2.4678 = 293.6

Example-6:

Find the number whose logarithm is – 2.4678.

Solution:

Let $\log N = -2.4678$ = –2 –1 + 1 – 0.4678 = – 3 + .5322 = 3.5322

From Antilog Table,

For mantissa 0 .532, the number = 3404.

For mean difference 2, the number = 2

$\therefore$For mantissa 0.5322, the number = (3404 + 2) = 3406

The characteristic is –3, therefore the number is less than one and there must be two zeros just after the decimal point.

Hence, antilog –2.4678 = 0.003406.

Example-7:

Find the value of (i) $\log_2 64$; (ii) $\log_3 \frac{1}{9}$; (iii) $\log_9 3$ (iv) $\log_8 0.25$

Solution:

(i) Let $\log_2 64 = x$

$or,\ 64 = 2^x$

$or,\ 2^6 = 2^x$

$\therefore x = 6$

(ii) Let $\log_3 \frac{1}{9} = x$

$or,\ \frac{1}{9} = 3^x$

$or,\ 9^{-1} = 3^x$

$or,\ 3^{-2} = 3^x$

$\therefore\ x = -2$

(iii) Let $\log_9 3 = x$

$or\ 3=9^x$

$or\ 3^1 = 3^{2x}$

$or\ 2x=1$

(iv) Let $\log_8 0.25 = x$

$or,\ 0.25 = 8^x$

$or,\ \frac{1}{4} = 2^{3x}$

$or,\ 4^{-1} = 2^{3x}$

$\therefore \ x = \frac{1}{2}$

$or,\ 2^{-2} = 2^{3x}$

$or,\ 3x = -2$

$\therefore \ x = -\ 2/3$

Example-8:

Find the logarithm of the following to the base indicated in brackets.

(i) 27, (3); (ii) 64, (8); (iii) 1000, (10); (iv) 0.25, (2).

Solution:

(i) $27 = 3^3$

$\therefore log_3 27 = 3.$

(ii) $64 = 8^2$

$\therefore log_8 64 = 2.$

(iii) $1000 = 10^3$

$\therefore \ log_{10} 1000 = 3.$

(iv) $0.25 = 2^{-2}$

$\therefore \ log_2 0.25 = -2.$

Example-9:

Without using tables, evaluate

$$log_{10} \frac{41}{35} + log_{10} 70 - log_{10} \frac{41}{2} + 2\ log_{10} 5$$

Solution:

$$log_{10} \left(\frac{41}{35} \times 70 \times \frac{2}{41} \times 5^2 \right)$$

$= log_{10} 100$

$= log_{10} 10^2$

$= 2\ log_{10} 10 = 2$

Example-10:

Simplify $7\ log \frac{10}{9} - 2\ log \frac{25}{24} + 3\ log \frac{81}{80}$

Solution:

$7\ log \frac{10}{9} - 2\ log \frac{25}{24} + 3\ log \frac{81}{80}$

$= 7\ [log\ 10 - log\ 9] - 2[log\ 25 - log\ 24] + 3\ [log\ 81 - log\ 80]$

$= 7[(log\ 5 + log\ 2) - log\ 3^2] - 2[log\ 5^2 - (log\ 3 + log\ 2^3)] + 3[log\ 3^4 - (log 5 + log 2^4)]$

$= 7\ log\ 5 + 7\ log\ 2 - 14\ log\ 3 - 4\ log\ 5 + 2\ log\ 3 + 6\ log\ 2 + 12\ log\ 3 - 3\ log\ 5 - 12\ log\ 2$

$= (7 - 4 - 3)\ log\ 5 + (2 - 14 + 12)\ log\ 3 + (7 + 6 - 12)\ log\ 2$

$= log 2.$

Example-11:

Find the value of $log\frac{75}{16} - 2\ log\frac{5}{9} + log\frac{32}{243}$, when 10 is the base of each logarithm.

Solution:

$log\frac{75}{16} - 2\ log\ \frac{5}{9} + log\frac{32}{243}$

$= [log_{10} 75 - log_{10} 16] - 2[log_{10} 5 - log_{10} 9] + [log_{10} 32 - log_{10} 243]$

$= [(log_{10} 5^2 + log_{10} 3) - log_{10} 4^2] - 2[log_{10} 5 - log_{10} 3^2] + [(log_{10} 4^2 + log_{10} 2) - log_{10} 3^5]$

$= 2\ log_{10} 5 + log_{10} 3 - 2\ log_{10} 4 - 2\ log_{10} 5 + 4\ log_{10} 3 + 2\ log_{10} 4 + log_{10} 2 - 5\ log_{10} 3$

$= log_{10} 2.$

Example-12:

Prove that,

$$(log\frac{3}{2}).(log\frac{4}{3}).(log\frac{5}{4}).(log\frac{6}{5}).(log\frac{7}{6}).(log\frac{8}{7}) = 3$$

Solution:

$$L.H.S. = \frac{\log 3 \times \log 4 \times \log 5 \times \log 6 \times \log 7 \times \log 8}{\log 2 \times \log 3 \times \log 4 \times \log 5 \times \log 6 \times \log 7}$$

$$= \frac{\log\ 8}{\log\ 2} = \frac{\log\ 2^3}{\log\ 2} = \frac{3\ \log\ 2}{\log\ 2} = 3$$

Therefore, *L.H.S = R.H.S.* (Proved).

Example-13:

Solve the equation:

$log_{10} (3x+2) - 2\ log_{10} x = 1 - log_{10} (5x - 3)$

Solution:

$log_{10} (3x+2) - 2\ log_{10}\ x = 1 - log_{10} (5x - 3)$

or, $log_{10} (3x+2) + log_{10} (5x - 3) - log_{10}\ x^2 = 1$

or, $log_{10} \left[\frac{(3x+2)(5x-3)}{x^2}\right] = 1$

$$\therefore \frac{(3x+2)(5x-3)}{x^2} = 10^1$$

or, $15x^2 - 9x + 10x - 6 = 10x^2$

or, $5x^2 + x - 6 = 0$

or, $5x^2 + x - 6 = 0$

or, $(5x+6)(x-1) = 0$

$\therefore x = 1$ *or* $-6/5$

since x cannot be negative, $x = 1$.

Example-14:

Show that $x^{\log y - \log z} . y^{\log z - \log x} . z^{\log x - \log y} = 1$

Solution:

Let the left side = N, then multiply both sides by *log*; we have

$\log N = \log (x^{\log y - \log z}) + \log (y^{\log z - \log x}) + \log (z^{\log x - \log y})$

$= (\log y - \log z) \log x + (\log z - \log x) \log y + (\log x - \log y) \log z$

$= \log y . \log x - \log z . \log x + \log z . \log y - \log x . \log y + \log x . \log z$

$- \log y . \log z$

$= 0$

Therefore $N = 10^0 = 1$

Hence *L. H. S = R. H. S.* (Proved).

Example-15:

Find the value of

$\log_{27} 49$; if $\log_{10} 3 = 0.4771$ and $\log_{10} 7 = 0.8451$

Solution:

Here $\log_{27} 49$ can be written (by changing base) as

$$\frac{\log_{10} 49}{\log_{10} 27} = \frac{\log_{10} 7^2}{\log_{10} 3^3} = \frac{2\log_{10} 7}{3\log_{10} 3} = \frac{2(0.8451)}{3(0.4771)} = \frac{1.6902}{1.4313} = 1.18$$

Example-16:

Prove that $\dfrac{\log_7 243}{\log_8 3 . \log_{49} 32} = 6$

Solution:

By changing all logarithms on LHS to the base 10 by using the formula, we get

$$log_7\ 243 = \frac{\log 243}{\log 7} = \frac{\log 3^5}{\log 7} = \frac{5\ \log 3}{\log 7}$$

$$log_8\ 3 = \frac{\log 3}{\log 8} = \frac{\log 3}{3\ \log 2}$$

$$log_{49} 32 = \frac{\log 32}{\log 49} = \frac{\log 2^5}{\log 7^2} = \frac{5\ \log 2}{2\ \log 7}$$

Here $\frac{\log_7 243}{\log_8 3 . \log_{49} 32}$ $= log_7 243 \div (log_8 3 \times log_{49} 32)$

$$= \frac{5\ \log 3}{\log 7} \times \frac{3\ \log 2}{\log 3} \times \frac{2\ \log 7}{5\ \log 2} = 6\ \textit{(Proved)}$$

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Define logarithm. Is there any distinction between natural and common logarithm?
2. What are the fundamental rules of logarithmic operations?
3. Find the value of $log_{10} 20 + log_{10} 30 - \frac{1}{2} log_{10} 36$
4. If $log_{10}2 = 0.3010$ and $log_{10}3 = 0.4717$;

   find (i) $log_{10} 25$; and (ii) $log_{10} 4.5$
5. If $\log_{\sqrt{8}} x = 3\frac{1}{3}$; find the value of x.
6. Evaluate $log \frac{31}{21} + log\ 49 - log\ 62 + log\ 27 - log_{30} 87$
7. If log a = 0.589; log b = 2.856 and log c = 1.963; find the value of

   $log \left(\frac{a^4 b^{\frac{1}{3}}}{c^2} \right)$
8. Find the value of $\frac{1}{3} log_{10} 125 - 2\ log_{10} 4 + log_{10} 32$.
9. If $log\ 3 = 0.4771$; $log\ 2 = 0.3010$ and $log\ 7 = 0.8451$, find the value of $log \frac{48}{91}$
10. If $log_{10} [98+ \sqrt{x^2 - 12x + 36}\,] = 2$, find the value of x.
11. Show that $log\ 2 + 16\ log \frac{16}{15} + 12\ log \frac{25}{24} + 7\ log \frac{81}{80} = 1$.
12. Solve $log_{10} (7x - 9)^3 + log_{10} (3x - 4)^3 = 3$.
13. Prove that $11\ log \frac{10}{9} - 3\ log \frac{25}{24} + 5\ log \frac{81}{80} = log\ 3$

Multiple Choice Questions (√ the appropriate answer)

1. If $a^x = b$, then

   (a) $\log_b x = a$ (b) $\log_a x = b$ (c) $\log_a b = x$

2. If $\log_a b = c$; then

   (a) $b^c = a$ (b) $a^c = b$ (c) $a^b = c$.

3. The value of $log_5\left(\frac{1}{625}\right)$ is

(a) 4 (b) – 4 (c) $\frac{1}{4}$

4. The value of $\log_{\sqrt{2}} 16$ is

(a) 4 (b) 8 (c) 16.

5. If $\log_8 x = \frac{2}{3}$, then the value of x is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 4.

6. The value of $[log\frac{3}{5} + log\frac{5}{36} + log\ 12]$ is equal to:

(a) log 5 (b) log 3 (c) 0.

7. If $log\ 2 = 0.3010$ and $5^x = 400$; then x is equal to:

(a) 2.40 (b) 3.72 (c) 4.36

8. The value of $[log\left(\frac{a^2}{bc}\right) + log\left(\frac{b^2}{ac}\right) + log\left(\frac{c^2}{ab}\right)]$ is

(a) 0 (b) 1 (c) abc

9. If $log_{10} 2x = 1$, the value of x is

(a) $\frac{1}{5}$ (b) 100 (c) 5

10. The characteristic in $log\ (6.7432 \times 10^{-5})$ is

(a) –5 (b) – 4 (c) 1

11. The Mantissa of log 3274 is .5150. The value of log 0.3274 is

(a) 0.5150 (b) 1.5150 (c) 1.5150

Lesson-2: Natural Logarithm and Antilogarithm

After studying this lesson, you should be able to

- Explain the natural logarithm;
- Explain antilogarithm;.
- Apply the principles of logarithm to solve the mathematical problems.

Nature of Natural Logarithm

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Logarithms to the base *'e'* are known as *natural logarithms*. The value of *'e'* may be calculated from the *'e' series*, where

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \text{ .... } \infty$$

[Here ! is factorial, where n! = n(n-1) (n-2) .... 0!]

Hence 4! = 4 × 3 × 2 × 1 × 0! (since 0! = 1)

Again, 6! = 6 × 5 × 4 × 3 × 2 × 1 × 0!

= 6 × 5!

From *'e' series*, the value of *'e'* is 2.71828.

Let, $e^x = N$

or, $log_e N = x$

When the base of logarithm is *'e'*, it may be expressed is *ln;*

i.e., $log_e N = ln\ N.$

Again, $log_{10} N = \frac{log_e N}{log_e 10}$ *(through change of base)*

or, $log\ N = \frac{InN}{In10}$

$\therefore In\ N = log\ N \times ln\ 10$

Again, $In\ 10 = \frac{\log 10}{\log e} = \frac{1}{\log e}$

$\therefore \text{In N} = \text{log N} \times \frac{1}{\log e}$

or, $log\ N = In\ N \times log\ e$

ng scientific ulator we can ly find the value of ased number.

Using scientific calculator we can easily find the value of *'e'* based number:

For example, $log_e 5 = 1.6094$

$log_e 0.5 = -0.6931$

$log_e 10 = 2.3025$

$$log_e e = In\, e = 1.$$

Let us take same examples.

Example-1:

Find the value of n, if $(1.08)^n = 3$.

Solution:

Given, $(1.08)^n = 3$
or, $In(1.08)^n = In\ 3$
or, $n\ In\ (1.08) = In\ 3$
$$n = \frac{\ln 3}{\ln(1.08)} = \frac{1.0986}{0.07696} = 14.27\ (App.)$$

Example-2:

Find the value of i, if $(1+i)^{12} = 2$

Solution:

Here $(1+i)^{12} = 2$

or, $In\ (1+i)^{12} = In\ 2$

or, $12\ In\ (1+i) = In\ 2$

or, $\text{In}\ (1+i) = \frac{\ln 2}{12} = \frac{0.6931}{12} = 0.0577$

or, $(1+i) = e^{0.0577} = 1.0594$

or, $i = 1.0594 - 1 = 0.0594$

$\therefore\ i = 0.0594$

Anti-logarithm

Let $log_a N = x$, then N is called the anti-logarithm of x to the base a and is written in short as $antilog_a\, x$.

Let $log_a N = x$, then N is called the anti-logarithm of x to the base a and is written in short as $antilog_a\, x$

If $log_a N = x$, then $N = antilog_a\, x$

For example, if $log\ 1000 = 3$, then $antilog\ 3 = 1000$

If $log\ 708 = 2.8500$, then $antilog\ 2.8500 = 708$.

Example-3:

Find the number whose logarithm is 1.7238

Solution:

Let the number is x

Therefore, $\log x = 1.7238$

or, $x = \text{antilog } 1.7238$

$\therefore\ x = 52.9420$ (by using calculator).

Example-4:

Find the value of (539.45×49.638)

Solution:

Let $x = 539.45 \times 49.638$

$$\log x = \log(539.45 \times 49.638)$$
$$= \log 539.45 + \log 49.638$$
$$= 2.3195 + 1.6981$$

or, $\log x = 4.4276$

$\therefore\ x = \text{Antilog } 4.4276 = 26{,}776.88$

Example-5:

Solve the equation $3^x \cdot 7^{2x+1} = 11^{x+5}$

Solution:

Taking logarithm of both sides, we have

$$x \log 3 + (2x+1) \log 7 = (x+5) \log 11$$

or, $x \log 3 + 2x \log 7 + \log 7 = x \log 11 + 5 \log 11$

or, $x \log 3 + 2x \log 7 - x \log 11 = 5 \log 11 - \log 7$

or, $x (\log 3 + 2 \log 7 - \log 11) = 5 \log 11 - \log 7$

$$\therefore\ x = \frac{5 \log 11 - \log 7}{\log 3 + 2 \log 7 - \log 11} = \frac{5.2070 - 0.8451}{0.4771 + 1.6902 - 1.0414}$$

$$= \frac{4.3619}{1.1259} = 3.87 \text{ (App.)}$$

Example-6:

Evaluate by using logarithm $\dfrac{61.92 \times 0.07046}{401.535}$

Solution:

Let $x = \dfrac{61.92 \times 0.07046}{401.535}$

Taking logarithm of both sides, we have

$$\log x = \log 61.92 + \log 0.07046 - \log 401.535$$
$$= 1.7918 + (-1.1521) - 2.6037$$

$= 1.7918 - 1.1521 - 2.6037$

$= -1.964$

$\therefore\ x = \text{antilog}\ (-1.964) = 0.01086.$

Example-7:

Evaluate by using logarithm:

(i) $\dfrac{(6.284)^3 . (624)^{\frac{1}{2}}}{\sqrt[4]{0.005}}$ (ii) $\sqrt[7]{\dfrac{1}{0.8176 \times 36.21}}$

Solution:

(i) Let $x = \dfrac{(6.284)^3 . (624)^{\frac{1}{2}}}{\sqrt[4]{0.005}}$

Taking logarithm of both sides, we have

$log\ x = 3\ log\ (6.284) + \frac{1}{2}\ log\ (624) - \frac{1}{4}\ log\ (0.005)$

or, $log\ x = 3\ [0.7982] + \frac{1}{2}\ [2.7952] - \frac{1}{4}\ (- 2.3010)$

or, $log\ x = 2.3946 + 1.3976 - 0.5753$

or, $log\ x = 3.7922 - 0.5753 = 3.2169$

$\therefore\ x = antilog\ (3.2169) = 1647.78.$

(ii) Let $x = \sqrt[7]{\dfrac{1}{0.8176 \times 36.21}}$

Taking logarithm of both sides, we have

$log\ x = log\left[\dfrac{1}{0.8176 \times 36.21}\right]^{\frac{1}{7}}$

or, $log\ x = \frac{1}{7}\ [log\ 1 - log\ 0.8176 - log\ 36.21]$

or, $log\ x = \frac{1}{7}\ [0 - 0.0875 - 1.5588]$

or, $log\ x = \frac{1}{7}\ [0.0875 - 1.5588]$

or, $log\ x = \frac{1}{7}\ [- 1.4713] = - 0.2102$

$\therefore\ x = antilog\ (- 0.2102) = 0.6163$

So, $x = 0.6163.$

Example-8:

Find the value of $\dfrac{(435)^3.(0.056)^{\frac{1}{2}}}{(380)^4}$

Solution:

Let $x = \dfrac{(435)^3.(0.056)^{\frac{1}{2}}}{(380)^4}$

Taking logarithm of both sides, we have

$\log x = 3 \log 435 + \frac{1}{2} \log 0.056 - 4 \log 380$

or, $\log x = 3 \times 2.6385 + \frac{1}{2} \times (-1.2518) - 4 \times 2.5798$

or, $\log x = 7.9155 - 0.6259 - 10.3192$

or, $\log x = -3.0296$

Hence, $x = \text{antilog}\,(-3.0296) = 0.0009341.$

Example-9:

Find the 7^{th} root of 0.00001427

Solution:

Let $x = 0.00001427$

Taking logarithm of both sides, we have

$\log x = \frac{1}{7} \log (0.00001427)$

or, $\log x = \frac{1}{7}(-4.8456)$

or, $\log x = -0.6922$

or, $x = \text{antilog}\,(-0.6922) = 0.2031$ *(App.)*

Example-10:

Find the value using logarithm, $\dfrac{628.24 \times 93.536}{3.786}$

Solution:

Let $x = \dfrac{628.24 \times 93.536}{3.786}$

Taking logarithm of both sides we get

$\log x = \log 628.24 + \log 93.536 - \log 3.786$

or, $\log x = (2.79813 + 1.97098 - 0.57818)$

or, $\log x = 4.19093$

$\therefore\ x = antilog\ 4.19093 = 15521.37.$

Example-11:

Solve $2^x.3^{2x} = 100$

Solution:

$2^x.3^{2x} = 100$

$or,\ log\ (2^x.3^{2x}) = log\ 100$

$or,\ x\ log\ 2 + 2x\ log\ 3 = log\ 10^2$

$or,\ x(0.30103) + 2x(0.47712) = 2\ log\ 10$

$or,\ 0.30103x + 0.95424x = 2$

$or,\ 1.2552x = 2$

$$\therefore\ x = \frac{2}{1.25527} = 1.59328.$$

Hence, $x = 1.59328$.

Example-12:

Solve $3^{2x} - 3^{x+1} + 2 = 0$

Solution:

$3^{2x} - 3^{x+1} + 2 = 0$

or, $(3^x)^2 - 3^x.3 + 2 = 0$

Let $3^x = y$

$\therefore\ y^2 - 3y + 2 = 0$

$or,\ y^2 - 2y - y + 2 = 0$

$or,\ y\ (y - 2) - 1(y - 2) = 0$

$or,\ (y - 2)\ (y - 1) = 0$

Now either, $y - 2 = 0$ or, $y - 1 = 0$

$or,\ y = 2$	$or,\ y = 1$
$or,\ 3^x = 2$	$or,\ 3^x = 1$
$or,\ log\ 3^x = log\ 2$	$or,\ log\ 3^x = log\ 1$
$or,\ x\ log\ 3 = log\ 2$	$or,\ x\ log\ 3 = log\ 1$
$or,\ x = \frac{\log 2}{\log 3}$	$or,\ x\ log\ 3 = 0$
$or,\ x = \frac{0.30103}{0.47712}$	$or,\ x = 0$
$or,\ x = 0.6309.$	

$\therefore$ *Hence x = 0 .6309 or 0.*

Example-13:

Prove that, $7 \log \frac{15}{16} + 6 \log \frac{8}{3} + 5 \log \frac{2}{5} + \log \frac{32}{25} = \log 3$

Solution:

L.H.S. $7 \log \frac{3 \times 5}{2^4} + 6 \log \frac{2^3}{3} + 5 \log \frac{2}{5} + \log \frac{2^5}{5^2}$

$= 7(\log 3 + \log 5 - 4 \log 2) + 6(3 \log 2 - \log 3) + 5(\log 2 - \log 5) + (5 \log 2 - 2 \log 5)$

$= 7 \log 3 + 7 \log 5 - 28 \log 2 + 18 \log 2 - 6 \log 3 + 5 \log 2 - 5 \log 5 + 5 \log 2 - 2 \log 5$

$= \log 3$ *(Proved)*

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Find the value of $(431.96)^{26}$.
2. Find the value of $\sqrt[6]{5896.31}$.
3. Find the value of $log_2\sqrt{6} + log_2\sqrt{2/3} - log\ 10$.
4. Find the value of $log_2\sqrt{3/2} + log_2\sqrt{5/3} - log_2\sqrt{5}$.
5. Find the value of x; if $log_4 x + log_2 x = 6$.
6. Evaluate $\frac{1002.76}{12\times 82}$ by using logarithm.
7. Solve $10^{4x-5} . 32^x = 5^{3-x} . 7^x$
8. Solve for x, if $log_x (8x - 3) - log_x 4 = 2$.
9. Evaluate $\frac{61.42\times 10.70}{401.53}$

Multiple Choice Questions (√ the appropriate answer)

1. $(log_{10} 40000 - log_{10} 4)$ is equal to:

   (a) 4 (b) 1000 (c) 39996

2. If $log\ (x+1) + log\ (x-1) = log\ 3$, then x is equal to

   (a) 7 (b) 2 (c) 8

3. If $log_{10} 125 + log_{10} 8 = x$; then x is equal to

   (a) –3 (b) 3 (c) 1/3

4. The value of x satisfying $log_{32} x = 0.80$ is:

   (a) 25.6 (b) 10 (c) 16.

5. If $\frac{loga}{b-c} = \frac{logb}{c-a} = \frac{logc}{a-b}$, then the value of $a^a b^b c^c$ is

   (a) abc (b) $\frac{1}{abc}$ (c) 1.