Applications to Economics and Business

12

This unit is designed to introduce the learners to the basic concepts associated with the applications of mathematics in business and economics. The learners will learn about different types of functions that are widely used in economics and the relationships among total, average and marginal functions. This unit also discusses the elasticity of demand and supply, consumers' surplus and producers' surplus etc. Some relevant examples are provided in this unit for clear understanding to the learners.

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Lesson-1: Uses of Different Functions in Business and Economics

After studying this lesson, you should be able to:

- Develop different functions relating to demand, supply, cost, revenue, profit and production;
- Analyze different types of functions;
- Determine total cost, average cost and marginal cost;
- Determine total revenue and marginal revenue;
- Determine total profit and marginal profit.

Introduction

Functions explain the nature of correspondence between variables indicated by some formula, graph or a mathematical equation. A function is a term used to symbolize relationship between or among the variables. When two variables are so related that for any arbitrarily assigned value to one of them there corresponds a definite value or a set of definite values for the other, the second variable is said to be the function of the first. We shall now introduce some different types of functions, which are particularly useful in business and economics.

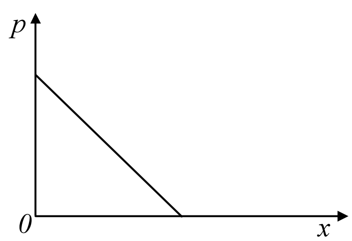
A function is a term used to symbolize relationship between or among the variables.

Demand Function

Demand functions are an essential concept in the study of economics. Usually these functions are curves rather than straight lines, but straight lines provide good illustrations of demand characteristics. The demand function specifies the amounts of a particular commodity that buyers are willing and able to purchase at each price in a series of possible prices during a specified period of time. Demand refers to the relationship between price and quantity. Conventionally, we plot price on the vertical axis and quantity demanded on the horizontal axis.

Demand refers to the relationship between price and quantity.

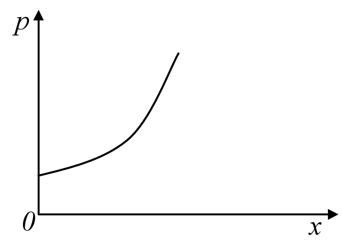
Let p be the price and x be the quantity demanded, the function $x = f(p)$ is plotted as a demand curve. It usually slopes downwards as demand decreases when price increases.



Supply Function

Supply function specifies the amounts of a particular product that a producer is willing and able to produce and make available for sale at each price in a series of possible prices during a specified period of time.

Let p be the price and x be the quantity supplied; the function $x = g(p)$ is plotted as a supply curve. When price increases quantity supplied increases, therefore, a supply curve slopes upward.



Total Revenue (TR)

A firm's total revenue that is derived from the sale of a product is given by price multiplied by quantity. It can be expressed mathematically, $TR = p \times q$.

From the demand function we observe that price is a function of quantity,

i.e., $p = f(q)$

Total revenue is thus represented as a function of quantity,

i.e., $TR = f(q) \times q$

Total Cost (TC)

> *The cost of production to the firm depends upon the costs of inputs used in the production process and the quantity of product manufactured.*

The cost of production to the firm depends upon the costs of inputs used in the production process and the quantity of product manufactured. If x is the quantity produced of a certain product by a firm at a total cost C, we can write the total cost function: $TC = f(x)$.

It may be noted that the total cost TC of producing goods can be analyzed into two parts: (i) fixed cost which is independent of x with certain limits, and (ii) variable cost depending on x. Thus, we may have cost function of the type, $TC = FC + VC$, where FC is fixed cost and VC is variable cost.

Cost curves are obtained from the knowledge of production functions. Usually, the cost curve is rising to the right as the cost of production generally increases with the output (x).

Total Profit

> *Profits of a firm depend upon both revenue and cost.*

Profits of a firm depend upon both revenue and cost. Profits are defined as the excess of total revenue over total costs. Symbolically, it can be expressed as, $P = TR - TC$.

Average Revenue (AR)

The average revenue from a product is found by dividing the total revenue by the quantity of the product sold. The function that describes average revenue is the quotient of the total revenue function and the quantity.

We see that average revenue function and the demand function are equivalent.

$$AR = \frac{TR}{q} = \frac{f(q).q}{q} = f(q)$$

Average Cost (AC)

Average cost of production or cost per unit is obtained by dividing total cost by the quantity produced.

$$AC = \frac{C}{x}$$

Marginal Revenue (MR)

Marginal revenue is the additional revenue derived from selling one more unit of a product or service. If each unit of a product sells at the same price, the marginal revenue is always equal to the price.

Marginal revenue is the additional revenue derived from selling one more unit of a product or service.

Thus, $MR = \dfrac{d(TR)}{dq}$; the rate of change of revenue with respect to units of output.

Marginal Cost (MC)

Marginal cost is defined as the change in total cost incurred in the production of an additional unit.

$MC = \dfrac{d(TC)}{dq}$; the rate of change of cost with respect to units of production.

Marginal Profit (MP)

Marginal profit analysis is concerned with the effect on profit if one additional unit of a product is produced and sold. As long as the additional revenue brought in by the next unit exceeds the cost of producing and selling that unit, there is a net profit from producing and selling that unit and total profit increases. If, however, the additional revenue from selling the next unit is exceeded by the cost of producing and selling the additional unit, there is a net loss from that next unit and total profit decreases.

Marginal profit analysis is concerned with the effect on profit if one additional unit of a product is produced and sold.

$MP = \dfrac{d(TP)}{dq}$; the rate of change of profit with respect to units of output.

A rule of thumb concerning whether or not to produce an additional unit (assuming profit maximization is of greatest importance) is given below.

(i) If $MR > MC$, produce the next unit.

(ii) If $MR < MC$, do not produce the next unit.

(iii) If $MR = MC$, the total profit will be maximized.

Production Function

A production function is a technical relationship between the inputs of production and the output of the firm's.

A production function is a technical relationship between the inputs of production and the output of the firm's. The relationship is such that the level of output depends upon the level of inputs used, not vice versa. A production function can be written as: $Q = f(L, K)$, where L and K are quantities of labor and capital respectively required to produce Q.

In Economics, the Cobb – Douglas production function defined as

$$Q = aL^{\alpha}K^{\beta}\text{, where } \alpha + \beta = 1.$$

Utility Function

Utility is the power of a commodity to satisfy human want.

The term utility refers to the benefit or satisfaction or pleasure of a person gets from the consumption of a commodity or service. In abstract sense, utility is the power of a commodity to satisfy human want, i.e., utility is want-satisfying power. A commodity is likely to have utility if it can satisfy a want. For example, bread has the power to satisfy hunger, water quenches our thirst and so on.

If $U(x, y)$ denotes the satisfaction obtained by an individual when he buys quantities x and y of two commodities X and Y respectively, then $U(x, y)$ is called the utility function or utility index of the individual.

Consumption Function

If C is the total consumption of the community dependent on income Y and propensity to consume c, the aggregate consumption function is defined by

$$C = a + cY$$

But since $Y = C + S$

$S = Y - (a + cY)$; This is the savings function of the community.

Relationship between Average Cost (AC) and Marginal Cost (MC)

Let us assume that total cost, $C = f(x)$

Thus, $AC = \dfrac{C}{x}$

$$\frac{d}{dx}(AC) = \frac{d}{dx}(\frac{C}{x}) = \frac{x\frac{dC}{dx} - C}{x^2} = \frac{1}{x}(\frac{dC}{dx} - \frac{C}{x}) = \frac{1}{x}(MC - AC)$$

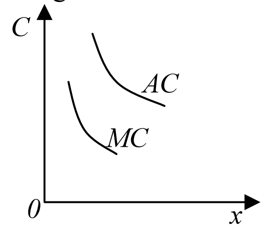
Case 1: When average cost curve slopes downwards, i.e., when AC is declining, its slope will be negative.

$$\frac{d}{dx}(AC) < 0$$

$$\frac{1}{x}(MC - AC) < 0$$

$$MC - AC < 0$$

MC < AC



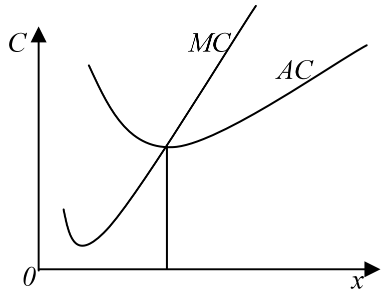
Thus when AC curves slopes downwards MC curve will lie below AC curve.

Case 2: When AC curve reaches a minimum point, its slope becomes zero.

$$\frac{d}{dx}(AC) = 0$$

$$\frac{1}{x}(MC - AC) = 0$$

MC – AC = 0
MC = AC



Thus MC curve and AC curve intersect at the point of minimum average cost.

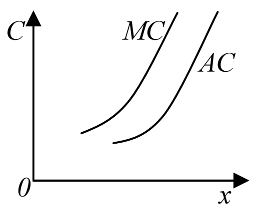
Case 3: When average cost curve slopes upward

$$\frac{d}{dx}(AC) > 0$$

$$\frac{1}{x}(MC - AC) > 0$$

$$MC - AC > 0$$

MC >AC



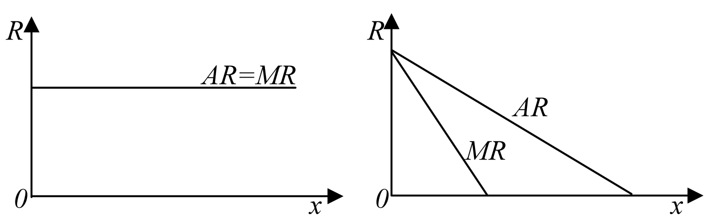
Thus when AC curve slopes upward MC curve will be above AC curve.

Relationship between Average Revenue (AR) and Marginal Revenue (MR)

When average revenue curve slopes downwards, i.e., when AR is declining, its slope will be negative. Thus when AR curves slopes downwards MR curve will lay below AR curve. This is common in imperfect competitive market.

When AR curves slopes downwards MR curve will lay below AR curve.

When AR is always same then there is no difference between AR and MR. This relationship will be applicable for pure competitive market.



Pure competitive market   Imperfect competitive market

Illustrative Examples:

Example-1:
Let the unit demand function be $x = ap + b$ and the cost function be $c = ex + f$, where x = sales (in units), p = price (in Tk.), f = fixed cost

(in Tk.), *e* = variable cost, *b* = demand, and when *p = 0, a* = slope of unit demand function.

Required: (i) find the cost *C* as a function of *p*.

(ii) find the revenue function *R (x)*.

(iii) find the profit function *P (x).*

Solution:

(i) Cost, $c = ex + f$

$$= e(ap + b) + f \,.$$

(ii) Revenue = price × quantity

$$= p \times x$$

$$= (\frac{x}{a} - \frac{b}{a}).x$$

$$= \frac{x^2}{a} - \frac{bx}{a}$$

(iii) Profit = Revenue – Cost

$$\text{P (x)} = (\frac{x^2}{a} - \frac{bx}{a}) - (ex + f)$$

$$= \frac{x^2}{a} - \frac{bx}{a} - ex - f$$

$$= \frac{x^2}{a} - (\frac{b}{a} + e)x - f \,.$$

Example-2:

Find the average cost and marginal cost if total cost

$$\text{TC} = 1000 + 100\text{q} - 10\text{q}^2 + \text{q}^3$$

Solution:

$$\text{Average cost (AC)} = \frac{\text{TC}}{\text{q}} = \frac{1000 + 100\text{q} - 10\text{q}^2 + \text{q}^3}{\text{q}}$$

$$= \frac{1000}{\text{q}} + 100 - 10\text{q} + \text{q}^2$$

$$\text{Marginal cost (MC)} = \frac{\text{d}}{\text{dq}}(\text{TC})$$

$$= \frac{\text{d}}{\text{dq}}(1000 + 100\text{q} - 10\text{q}^2 + \text{q}^3)$$

$$= 100 - 20\text{q} + 3\text{q}^2$$

Example-3:

If the demand function of the monopolist is $3q = 98 - 4p$ and average cost is $3q + 2$ where q is output and p is the price, find maximum profit of the monopolist.

Solution:

Given average cost, AC = $3q+2$

Total Cost, TC = AC × q = $(3q+2).\, q = 3q^2+2q$

Marginal cost (MC) = $\frac{d}{dq}(TC)$

$$\text{MC} = \frac{d}{dq}(3q^2+2q) = 6q+2$$

Again given that, $3q = 98-4p$

$$p = \frac{98-3q}{4}$$

Total Revenue (TR) = price × quantity

$$\text{TR} = p \times q$$

$$\text{TR} = \frac{98-3q}{4}.q = \frac{98q-3q^2}{4}$$

Marginal Revenue (MR) = $\frac{d}{dq}(\frac{98q-3q^2}{4})$

$$= \frac{98}{4} - \frac{6q}{4}$$

We know that under monopoly market, profit will be maximum at MC = MR.

$$6q+2 = \frac{98}{4} - \frac{6q}{4}$$

$$24q+8 = 98-6q$$

$$30q = 90$$

$$q = 3.$$

So the maximum profit of the monopolist will be obtained at q = 3.

Again Total Profit (TP) = TR – TC

$$\text{TP} = (\frac{98q-3q^2}{4}) - (3q^2+2q)$$

When q = 3, then the profit will be maximum

i.e., maximum profit = $\frac{(98)(3)-3(3)^2}{4} - 3(3)^2 - 2(3) = 33.75$

Example-4:

Show that marginal cost (MC) must equal marginal revenue (MR) at the profit maximizing level of output.

Solution:

We know that, Total profit = Total revenue – Total cost

i.e., TP = TR – TC

To maximize TP, $\frac{d(TP)}{dQ}$ must equal zero.

$$\frac{d(TP)}{dQ} = \frac{d(TR)}{dQ} - \frac{d(TC)}{dQ} = 0$$

$$\frac{d(TR)}{dQ} = \frac{d(TC)}{dQ}.$$

MR = MC (showed)

Example-5:

If the cost function is $C(x) = 4x + 9$ and the revenue function is $R(x) = 9x - x^2$, where *x* is the number of units produced (in thousands) and *R* and *C* are measured in million of Tk., find the following:

(i) Marginal revenue.
(ii) Marginal revenue at $x = 5$.
(iii) Marginal cost.
(iv) The fixed cost.
(v) The variable cost at $x = 5$.
(vi) The break-even point, that is $R(x) = C(x)$.
(vii) The profit function.
(viii) The most profitable output.
(ix) The maximum profit.
(x) The marginal revenue at the most profitable output.
(xi) The revenue at the most profitable output.
(xii) The variable cost at the most profitable output.

Solution:

Given that, $R(x) = 9x - x^2$

$C(x) = 4x + 9$

(i) $MR = \frac{d}{dx}(R) = \frac{d}{dx}(9x - x^2) = 9 - 2x$

(ii) When $x = 5$, then $MR = 9 - 2 \times 5 = -1$.

(iii) $MC = \frac{d}{dx}(C) = \frac{d}{dx}(4x + 9) = 4$

(iv) The fixed cost, FC = 9.

(v) When $x = 5$, the variable cost (VC) is $(4 \times 5) = 20$.

(vi) For break-even point, *R*(*x*) = *C*(*x*).

$$9x - x^2 = 4x + 9$$

$$x^2 - 5x + 9 = 0$$

$$x = \frac{5 \pm \sqrt{-11}}{2}.$$

(vii) Profit, P = R – C = $\{(9x - x^2) - (4x + 9)\} = 5x - x^2 - 9$.

(viii) Here profit, $P = 5x - x^2 - 9$

$$\frac{dP}{dx} = 5 - 2x$$

For maximum or minimum, $\frac{dP}{dx} = 0$

$$5 - 2x = 0$$

$$x = \frac{5}{2}.$$

Again $\frac{d^2P}{dx^2} = -2$; which is negative.

So the profit function is maximum at $x = \frac{5}{2}$.

Thus, the required most profitable output is $x = \frac{5}{2}$.

(ix) The maximum profit, $P = 5x - x^2 - 9$.

$$= 5 \times \frac{5}{2} - (\frac{5}{2})^2 - 9;$$

$$= -\frac{11}{4}, \text{ which shows a loss.}$$

(x) When $x = \frac{5}{2}$, then $MR = 9 - 2x = 9 - 2 \times \frac{5}{2} = 4$.

(xi) When $x = \frac{5}{2}$, then $R = 9x - x^2 = 9 \times \frac{5}{2} - (\frac{5}{2})^2 = 16.25$.

(xii) When $x = \frac{5}{2}$, then variable cost $VC = 4x = 4 \times \frac{5}{2} = 10$.

Questions for Review:

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. The total cost C for output x is given by $C = \frac{2}{3}x + \frac{35}{2}$.

   Find (i) Total cost when output is 50 units.
   (ii) Average cost when output is 100 units.
   (iii) Marginal cost when output is 60 units.

2. The total cost of a firm is given by $C = 0.04q^3 - 0.9q^2 + 10q + 100$

   Find (i) Average cost
   (ii) Marginal cost
   (iii) Slope of average cost
   (iv) Slope of marginal cost
   (v) Value of q at which average variable cost is minimum.

3. The unit demand function is $x = \frac{25-2p}{3}$ where x is the number of units and p is the price. Assume that average cost per unit is Tk.50.

   Find (i) The revenue function R in terms of price P.
   (ii) The cost function C.
   (iii) The profit function P.
   (iv) The price per unit that maximizes the profit function
   (v) The maximum profit.

4. The marginal cost function (y) for production (x) is $y = 20 + 48x - 6x^2$ if the total cost of producing one unit is \$50. Find the total cost function and average cost function.

5. The anticipated price of a product is Tk.15. The fixed cost of manufacturing the product is Tk.10,000 and the variable cost is Tk.10 per unit. Develop revenue, total cost function and calculate breakeven production. Calculate revenue, total cost, and profit at the breakeven product.

Lesson-2: Elasticity

After studying this lesson, you should be able to:

- Define the concept of elasticity, elasticity of demand, and elasticity of supply.
- Describe the techniques of measuring elasticity.

Elasticity

Elasticity is the ratio that measures the responsiveness or sensitiveness of a dependent variable to the changes in any of the independent variables. More clearly, the term elasticity refers to the percentage change in dependent variable divided by the percentage change in independent variable.

The term elasticity refers to the percentage change in dependent variable divided by the percentage change in independent variable.

Thus, elasticity = $\dfrac{\text{Percentage change in dependent variable}}{\text{Percentage change in independentvariable}}$

If $y = f(x)$, i.e., y depends on x, then the elasticity of y with respect to x is

Elasticity of y = $\dfrac{\text{Percentage change in } y}{\text{Percentage change in } x} = \dfrac{\%\,\Delta y}{\%\,\Delta x}$.

Elasticity of Demand

The elasticity of demand is the measure of responsiveness of demand for a commodity to the changes in any of its determinants. The determinants of demand are the commodity's own price, income, price of related goods (substitutes and complements) and consumers' expectations regarding future price.

The elasticity of demand is the measure of respon-siveness of demand for a commodity to the changes in any of its determinants.

$$Q_X^D = f(P_x, M, P_y, P_z ....)$$

where Q_X^D = quantity demanded of commodity x.

P_x = price of commodity x

M = money income of the consumer

P_y = price of the substitute, x and y are substitute to each other

P_z = price of complement, x and z are complement to each other.

Price Elasticity of Demand

The average price elasticity of demand is the proportionate response of quantity demanded to the change in price. Let δp be small change in price p and δx be a small change in the quantity demanded.

The average price elasticity of demand is the proportionate response of quantity demanded to the change in price.

The average price elasticity of demand = $\dfrac{\frac{\delta x}{x}}{\frac{\delta p}{p}} = \dfrac{p}{x}.\dfrac{\delta x}{\delta p}$

Since the point elasticity of demand is the limiting value of average price elasticity. So, the point elasticity of demand is $E_d = \frac{p}{x} . \frac{dx}{dp}$.

Generally, slope of the demand curve is negative and thus E_d is negative, i.e.,

$$|E_d| = -\frac{p}{x} . \frac{dx}{dp}.$$

Thus, when $|E_d| > 1$, the demand is elastic.

when $|E_d| < 1$, the demand is inelastic.

when $|E_d| = 1$, the demand is unitary elastic.

Price Elasticity of Supply

The price elasticity of supply measures the responsiveness of the quantity supplied of a commodity to a change in its price.

The price elasticity of supply measures the responsiveness of the quantity supplied of a commodity to a change in its price.

Price elasticity of supply = $\frac{\text{Percentage change in quantity supplied}}{\text{Percentage change in price}} = \frac{\%\,\Delta Q}{\%\,\Delta P}$

Thus, the price elasticity of supply is denoted by e_s and is given by $\frac{p}{x} . \frac{dx}{dp}$; where x is quantity supplied and p is price.

Income Elasticity of Demand

The income elasticity of demand is a measure of the responsiveness of quantity demanded to a change in income.

The income elasticity of demand is a measure of the responsiveness of quantity demanded to a change in income, other things remaining the same. It is calculated by using the following formula:

Income elasticity of demand =

$$\frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in income}}.$$

It is denoted by E_y and is given by $\frac{y}{x} . \frac{dx}{dy}$ where x is the quantity demanded and y is the income per head in the relevant group of people.

The income elasticity of demand may be positive or negative. This motivates the definition of following types of goods.

Case 1: Goods are Luxury, if $E_y > 1$.

Case 2: Goods are Necessity of life, if $0 < E_y < 1$.

Case 3: Goods are Inferior, if $E_y < 0$.

Cross Elasticity of Demand

The cross elasticity of demand is a measure of the responsiveness of quantity demanded to the price of its substitutes or complements.

The cross elasticity of demand is a measure of the responsiveness of quantity demanded to the price of its substitutes or complements.

If x and y are related goods, the cross elasticity of demand,

$$E_{xy} = \frac{\frac{dQ_x}{Q_x}}{\frac{dP_y}{P_y}} = \frac{dQ_x}{dP_y}.\frac{P_y}{Q_x}.$$

When x and y are complementary goods, the sign of the cross elasticity is negative and if x and y are substitute goods, then the sign is positive.

Illustrative Examples:

Example-1:

Find the elasticity of demand for the function $p = 100 - x - x^2$.

Solution:

$$|E_d| = -\frac{p}{x}.\frac{dx}{dp}$$

$$= -\frac{p}{x}.\frac{1}{\frac{dx}{dp}} \quad = -\frac{p}{x}.\frac{1}{-1-2x}$$

$$= \frac{p}{x+2x^2} \quad = \frac{100 - x - x^2}{x(1+2x)}$$

Example-2:

The demand function is $Q = 20 - 5P$. Find the inverse function and estimate the elasticity at $P = 2$.

Solution:

Given that $Q = 20 - 5P$.

The inverse function is $P = 4 - 0.2Q$

$$\frac{dQ}{dP} = -5$$

At P = 2, Q = 10.

$$|E_d| = -\frac{\text{dQ}}{\text{dP}}.\frac{P}{Q} = -(-5).\frac{2}{10} = 1.$$

Example-3:

If the demand function is $p = 4 - 5x^2$; for what value of x, the elasticity of demand will be unity.

Solution:

The given demand function is $p = 4 - 5x^2$.

$$\frac{dp}{dx} = -10x.$$

The price elasticity of demand is $|E_d| = -\frac{p}{x}.\frac{dx}{dp}$

$$= -\frac{4-5x^2}{x}.\frac{-1}{10x} = \frac{4-5x^2}{10x^2}.$$

The elasticity of demand will be unity if $|E_d| = 1$.

$$\frac{4-5x^2}{10x^2} = 1.$$

$$10x^2 = 4 - 5x^2$$

$$15x^2 = 4$$

$$\therefore\ x = \frac{2}{\sqrt{15}}.$$

Example-4:

Find the elasticity of demand and supply at equilibrium price for demand function $p = \sqrt{100 - x^2}$ and supply function $x = 2p - 10$, where p is price and x is quantity.

Solution:

Equilibrium conditions can be determined by equating demand and supply.

$$\sqrt{100 - x^2} = \frac{x+10}{2}$$

or, $2\sqrt{100 - x^2} = x + 10$
or, $4\ (100 - x^2) = x^2 + 20x + 100$
or, $5x^2 + 20x - 300 = 0$
or, $x^2 + 4x - 60 = 0$
or, $(x + 10)\ (x - 6) = 0$
or, $x = 6$; (since negative quantity is not admissible).
$\therefore\ x = 8$.

Price elasticity of demand:

$$p = \sqrt{100 - x^2}$$

$$\frac{dp}{dx} = \frac{1}{2}.(100 - x)^{-½}.(-2x) = -\frac{x}{\sqrt{100 - x^2}}$$

$$|E_d| = -\frac{p}{x}.\frac{dx}{dp} = -\frac{8}{6}.\frac{-x}{\sqrt{100 - x^2}}$$

$$\therefore\ |E_d| = \frac{16}{9}$$

Price elasticity of supply:

Here $x = 2p - 10$

$$\frac{dx}{dp} = 2$$

Price elasticity of supply, $E_S = \frac{p}{x}.\frac{dx}{dp} = \frac{8}{6}.2 = \frac{8}{3}$

Questions for Review:

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. If the demand function is $Q = 1400 - P^2$. Find the price elasticity of demand at $P = 20$.
2. Find the elasticity of demand when demand function is $q = 7 - 2p$ at $p = 2$.
3. Find the elasticity of supply when supply function is $q = 2p^2 + 5$ at $p = 1$.

Lesson-3: Consumers' Surplus and Producers' Surplus

After completing this lesson, you will be able to:

- Explain consumers' surplus;
- Explain producers' surplus;
- Determine consumers' surplus;
- Determine producers' surplus.

Introduction

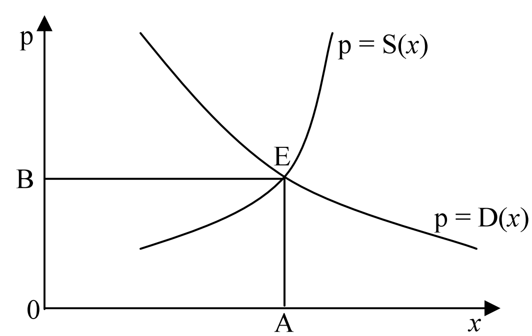
In the economic model of pure competition, it is assumed that all consumers of product pay the same price per unit of a product.

In the economic model of pure competition, it is assumed that all consumers of product pay the same price per unit of a product. This price comes about by the interplay of competitive market forces and is the price per unit at which the quantity of product consumers are willing and able to buy is matched by the quantity producers are willing and able to supply. The purpose of this lesson is to illustrate that this competitive situation benefits both consumer and supplier and to develop a measure of these benefits.

Market Equilibrium Position

Suppose the price p that a consumer is willing to pay for a quantity x of a particular commodity is governed by the demand curve $p = D(x)$. Further, suppose the price p that a producer is willing to charge for a commodity x of a particular commodity is governed by the supply curve $p = S(x)$. The point of intersection of the demand curve and the supply curve is called the equilibrium point E.

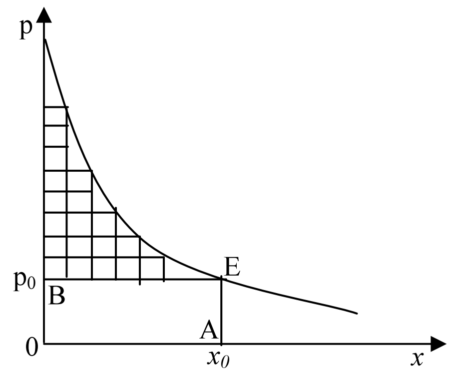
If the coordinates of E are (x_0, p_0), then p_0 is the market price a consumer is willing to pay for and a producer is willing to sell for a quantity x_0, the demand level of the commodity.



Consumer's Surplus (CS)

Difference between what consumers actually pay and the maximum amount that they would be willing to pay is called consumers' surplus.

A demand function represents the different prices consumers are willing to pay for different quantities of a good. In a free market economy, when some consumers' would be willing to pay more than the market equilibrium price for the commodity, this benefit to the consumers, i.e., difference between what consumers actually pay and the maximum amount that they would be willing to pay is called consumers' surplus.

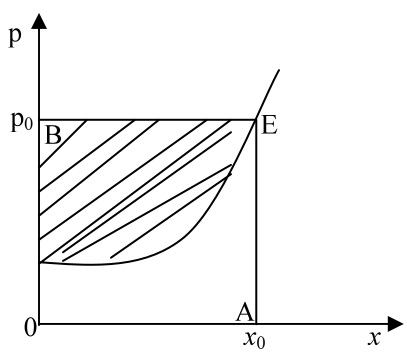


Thus, consumers' surplus = {total area under the demand curve *D(x)* from $x = 0$ to $x = x_0$} – {the area of the rectangle OAEB}.

Mathematically, Consumers' surplus = $\int_0^{x_0} D(x)dx - x_0 p_0$

Producer's Surplus (PS)

The relationship between the market price and the quantities producers are willing to supply is expressed as a supply function. The supply curve slopes upward to the right as because producers are willing to supply more at higher prices than at lower prices.



In a free market economy, when some producers would be willing to sell at a price below the market price p_0 that the consumers actually pays, the benefit of this to the producer, i.e., the difference between the revenue producers actually receive and what they have been willing to receive is known as producer's surplus (PS).

The difference between the revenue producers actually receive and what they have been willing to receive is known as producer's surplus (PS).

Thus, producers' surplus = {area of the rectangle OAEB} – {area below the supply curve from $x = 0$ to $x = x_0$}.

Mathematically, Producers' surplus = $x_0 p_0 - \int_0^{x_0} S(x)dx$

Illustrative Example:

Example-1:

The demand law for a commodity is $P = 1200 - q^2$. Find the consumer's surplus (CS) when the demand is15.

Solution:

Here given that $P = f(q) = 1200 - q^2$

If $q_0 = 15, P_0 = 1200 - 15^2 = 975$.

We know that consumer's surplus (CS) = $\int_0^{q_0} dq - p_0 q_0$

$$= \int_0^{15} (1200 - q^2)\text{dq} - 975 \times 15$$

$$= \left[1200q - \frac{q^3}{3}\right]_0^{15} - 14625.$$

$$= \left[1200 \times 15 - \frac{15^3}{3}\right] - 14625$$

= 2250.

Example-2:

The demand and supply functions under perfect competition for a product are $D(x) = 16 - x^2$ and $S(x) = 4 + x$ respectively. Find the market price, consumer's surplus (CS) and producer's surplus (PS).

Solution:

Here given that, demand function = $D(x) = 16 - x^2$ (i)

and supply function = $S(x) = 4 + x$ (ii)

Solving (i) and (ii) we get, $x = 3 = x_0$

When $x = 3$, $y = 7 = y_0$

We know that consumer's surplus (CS) = $\int_0^{x_0} D(x)dx - p_0 x_0$

$$= \int_0^{3} (16 - x^2)dx - 7 \times 3$$

= 18

Producer's surplus (PS) = $p_0 x_0 - \int_0^{x_0} S(x)dx$

$$= 7 \times 3 - \int_0^{3} (4 + x)dx$$

= 4.5

Example-3:

The demand and supply functions under perfect competition for a product are $D(x) = 16 - x^2$ and $S(x) = 2x^2 + 4$ respectively. Find the market price, consumer's surplus (CS) and producer's surplus (PS).

Solution:

Here given that, demand function = $D(x) = 16 - x^2$ (i)

and supply function = $S(x) = 2x^2 + 4$ (ii)

Solving (i) and (ii) we get, $x = 2 = x_0$

When $x = 2$, $y = 12 = y_0$

We know that consumer's surplus (CS) = $\int_0^{x_0} D(x)dx - p_0x_0$

$= \int_0^2 (16 - x^2)dx - 2X12$

$= 5.33$

Producer's surplus (PS) = $p_0x_0 - \int_0^{x_0} S(x)dx$

$= 2 \times 12 - \int_0^2 (2x^2 + 4)dx$

$= 10.67.$

Example-4:

The demand and supply functions under pure competition for a product are $D(q) = 25 - q^2$ and $S(q) = 2q + 1$ respectively. Find the market price, consumer's surplus (CS) and producer's surplus (PS).

Solution:

For market equilibrium, Demand (D) = Supply (S).

$$25 - q^2 = 2q + 1$$

Here, q = 4, or –6

$\therefore$ $q_0 = 4$ (since q cannot be negative)

and $p_0 = 9$

We know that consumer's surplus (CS) = $\int_0^{q_0} D(q)dq - p_0q_0$

$= \int_0^4 (25 - q^2)dq - 4 \times 9$

$= 42.67$

Producer's surplus (PS) = $p_0q_0 - \int_0^{q_0} S(q)dq$

$= 4 \times 9 - \int_0^4 (2q + 1)dq$

$= 16$

Example-5:

The demand and supply functions under pure competition for a product are $D(x) = 113 - q^2$ and $S(x) = (q+1)^2$ respectively. Find the market price, consumer's surplus (CS) and producer's surplus (PS).

Solution:

For market equilibrium, Demand (D) = Supply (S).

$$113 - q^2 = (q+1)^2$$

Here q = 7, or –8

$q_0 = 7$ (since q cannot be negative)

$p_0 = 64$

We know that consumer's surplus (CS) = $\int_0^{q_0} D(q)\,dq - p_0 q_0$

$= \int_0^{7} (113 - q^2)\,dq - 64 \times 7$

= 228.67

Producer's surplus (PS) = $p_0 q_0 - \int_0^{q_0} S(q)\,dq$

$= 64 \times 7 - \int_0^{7} (q+1)^2\,dq$

= 277.67

Example-6:

Under a monopoly the quantity sold and market price are determined by the demand function. If the demand function for a profit-maximizing monopolist is $P(Q) = 274 - Q^2$ and $MC = 4 + 3Q$, find the consumer's surplus (CS).

Solution:

Given $P(Q) = 274 - Q^2$

TR = $PQ = (274 - Q^2) \times Q = 274Q - Q^3$

$MR = 274 - 3Q^2$

The monopolist maximize profit at MR = MC

$$274 - 3Q^2 = 4 + 3Q$$

$Q_0 = 9$ and $P_0 = 193$

We know that consumer's surplus (CS) = $\int_0^{Q_0} D(Q)\,dQ - P_0 Q_0$

$= \int_0^{9} (274 - Q^2)\,dQ - 193 \times 9$

= 486 units.

Example-7:

The demand and supply functions are $D(x) = (12 - 2x)^2$ and $S(x) = 56 + 4x$ respectively. Determine consumer's surplus (CS) under monopoly (so as to maximize the profit) and the supply function is identified with the marginal cost function.

Solution:

$$TR = x \times D(x) = x(12 - 2x)^2 = x(144 - 48x + 4x^2)$$
$$= 144x - 48x^2 + 4x^3$$

$$MR = 144 - 96x + 12x^2$$

Since supply price is identified with *MC*, we have

$$MC = 56 + 4x$$

In order to find consumer's surplus (CS) under monopoly, i.e., to maximize the profit,

we have *MR* = *MC*

$$144 - 96x + 12x^2 = 56 + 4x$$

or, $3x^2 - 25x + 22 = 0$

$\therefore$ $x = 1$ or $x = \frac{22}{3}$

When $x_0 = 1$, $D(x_0) = P_0 = (12 - 2)^2 = 100$

We know that consumer's surplus (CS) = $\int_0^{x_0} D(x)dx - p_0 x_0$

$$= \int_0^1 (12 - 2x)^2 dx - 1 \times 100$$

$$= \frac{64}{3} \text{ units}$$

Again, when $x_0 = \frac{22}{3}, D(x_0) = P_0 = (12 - \frac{44}{3})^2 = \frac{64}{9}$

And consumer's surplus (CS) = $\int_0^{x_0} D(x)dx - p_0 x_0$

$$= \int_0^{22/3} (12 - 2x)^2 dx - \frac{22}{3} \times \frac{64}{9}$$

$$= \frac{19360}{81} \text{ units.}$$

Questions for Review:

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. The demand law for a commodity is $P = 20 - D - D^2$. Find the consumer surplus when the demand is 3.

2. The supply and demand functions for a product are $S(x) = 3x + 9$ and $D(x) = 30 - 4x$ respectively, where x represents units of quantity. Compute consumer's surplus and producer's surplus.

3. Determine consumer's surplus and producer's surplus if $S(q) = 10 - \frac{5}{q+1}$ and $D(q) = 8 + \frac{15}{q+1}$.