Module 3

Quantitative demand analysis

Introduction

The term *elasticity* refers to the number of units of a particular product consumers are willing to buy under specifically formulated conditions. In this module, we examine demand theory, or the forces that determine the demand for a firm's product, as well as the concept of elasticity. Demand is a function of a number of independent variables or determinants. In this module, we express demand as a mathematical expression. This will facilitate determining the demand elasticity with respect to a whole set of variables such as price, income and adverting. Measurements of elasticity can help business managers to understand the market characteristics of the goods or services they are selling, and thus can help them in planning their marketing strategies, especially the pricing of their products.

Upon completion of this module you will be able to:

Outcomes

- *explain* the concept of elasticity.
- *distinguish* between the point and the arc elasticity of demand and apply this concept to various decision-making situations.
- *demonstrate* the business application of income and other elasticity.
- *explain* the relationship between the price elasticity and pricing decisions.
- *demonstrate* the relationship between elasticity, marginal and total revenue.

Terminology

Advertisement elasticity:	Measures the responsiveness of sales to changes in the amount spent on advertising and promotion.
Cross elasticity:	Measures the responsiveness of sales of a product to changes in price of another product. Cross-price elasticity is a measure of the responsiveness of consumers to changes in the price of a particular good, Good X, relative to changes in the price of substitute or complementary products, Good Y. It provides a measure of the degree of substitutability or complementarity between product X and product Y.
Elasticity:	The percentage change in the dependent variable Y that is caused by a one per cent change in the

independent variable X

Income elasticity: Measures the responsiveness of sales to changes in consumer income. The percentage change in the quantity demanded is compared with the percentage change in income.

Price elasticity: Measures the responsiveness of sales of a product or service to changes in its price.

Elasticity of demand

If we lower a product's price, we know that sales will increase, but by how much? And what will happen to total revenue? What will happen to sales if consumers' disposable income increases? What will happen to sales if the advertising budget is increased? Will a change in the price of butter affect the sales of margarine? If so, by how much? These are important questions in the business world, and they all can be answered by understanding the concept and measurement of *elasticity*.

In general, the elasticity of any function is defined as the percentage change in the dependent variable Y that is caused by a one per cent (or relatively small) change in the independent variable X while all other independent variables are held constant. This general concept of elasticity is applicable to any function. In theory, the demand function has elasticity for each of its many independent variables. However, we shall confine our discussion to the four demand elasticities that are most widely discussed in the literature of demand theory. These are:

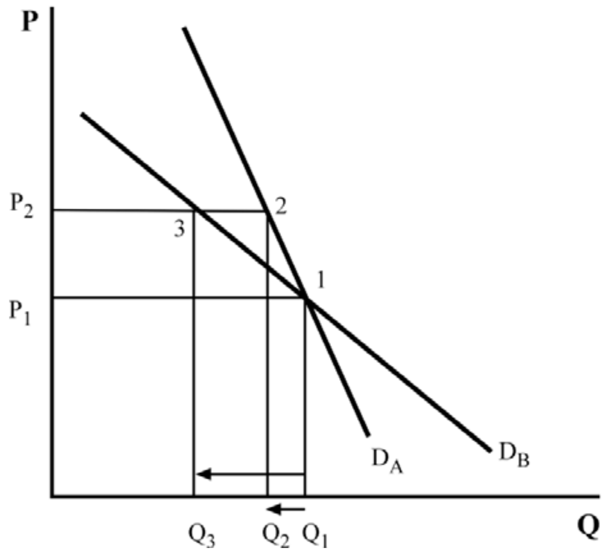
1. *price elasticity of demand*, which measures the responsiveness of sales to changes in price;
2. *income elasticity of demand*, which measures the responsiveness of sales to changes in consumer income;
3. *cross elasticity of demand*, which measures the responsiveness of sales of a product to changes in price of another price; and
4. *advertising elasticity*, which measures the responsiveness of sales to changes in the amount spent on advertising and promotion.

The (own) price elasticity of demand

Look at a firm that is considering a price increase. The firm understands that according to the law of demand, the increase in price will result in the loss of some sales. This may be acceptable if the loss in sales is not too large. If sales do not suffer much, the firm may actually increase its sales revenue when it raises its price. If sales drop substantially, however, sales revenues may decline and the firm could be worse off.

Figure 3-1 illustrates the implications of the firm's pricing decision when its demand curve has one of two alternative shapes, D_A and D_B. Suppose the firm is currently charging P_1 and selling Q_1, and is considering an increase in price to P_2.

Figure 3-1 Effect of pricing with two demand curves



When the demand curve is D_A, a change in price from P_1 to P_2 has only a small effect on the quantity demanded. However, when the demand curve is D_B, the same change in price results in a large drop in quantity demanded. When D_A is the demand curve, we would hypothesise that the increase in price would increase sales revenues since the price increase swamps the quantity decrease. Here, the quantity demanded is not very sensitive to price. But when D_B is the demand curve, the price increase would reduce sales revenues since the price increase is swamped by the quantity decrease. Here, the quantity demanded is very sensitive to price. We would expect that the price increase would decrease sales revenues.

As this analysis shows, the shape of the demand curve can strongly affect the success of the firm's pricing strategy. The concept of the own-price elasticity of demand summarises this effect by measuring the sensitivity of quantity demanded to price. The own-price elasticity of demand, commonly denoted by E_p, is the percentage change in quantity brought about by a 1 per cent change in price. Letting subscript '1' represent the initial situation and '2' represent the situation after the price changes, the formula for elasticity is:

$$E_P = \frac{\%\Delta inQ}{\%\Delta inP} = \frac{Q_2 - Q_1}{Q_1} / \frac{P_2 - P_1}{P_1} \qquad (1)$$

where Δ (the Greek letter delta) is the conventional symbol for 'change.' Suppose the price increases, in Figure 3-1, from P_1 (=\$5) to P_2 (=\$6). In case of D_B, the quantity demanded drops to 1,500 from 2,000, whereas in case of D_A, it drops to 1,800. Then for D_B,

$$E_P = \frac{(1500-2000)}{2000} / \frac{6-5}{5} = -\frac{500}{2000} / \frac{1}{5} = -\frac{5}{4} = -1.25 \qquad (2)$$

In this case, percentage decrease in quantity is 33 per cent whereas the percentage increase in price is 20 per cent. Using the same formula for the second demand curve, D_A, we see that the decrease in price from $6 to $5 indicates an elasticity coefficient of –0.50:

$$E_P = \frac{(1800-2000)}{2000} / \frac{6-5}{5} = -\frac{200}{2000} / \frac{1}{5} = -\frac{5}{10} = -0.50 \qquad (3)$$

The percentage increase in quantity in this case would be 10 per cent, (1800 – 1000)/2000. Thus, over the range of prices between $5.00 and $6.00, quantity demanded falls at a rate of 1.25 per cent in case of D_B, and at the rate of 0.50 in case of D_A.

The demand is said to be *elastic*, if the percentage change in the quantity demanded exceeds the percentage change in price; the elasticity coefficient E_p will have a value greater than one, which is the situation along demand curve, D_B. If the percentage change in the quantity demanded is less than the percentage change in price, then E_p will be less than one. This, by definition, is an *inelastic* demand, which is the situation along demand curve D_A. If E_p is equal to one, demand is unitary elastic.

Note that the minus sign in front of the coefficient is due to the fact that ΔQ and ΔP have opposite signs, that is, price, and quantity move in opposite directions along a demand curve. For the purposes of our analysis, however, we drop the negative sign and consider only the absolute value of the coefficient. These points are summarised in Table 3-1 below.

Table 3-1: Different types of elasticity

Demand responsiveness may be classified in *absolute* terms as:		
a.	perfectly elastic	Ep = ∞(infinity)
b.	Elastic	Ep >1
c.	unitarily elastic	Ep = 1
d.	Inelastic	Ep <1
e.	perfectly inelastic	Ep = 0

Price elasticity can be estimated using statistical techniques, and economists and marketers have estimated price elasticity for many products. But in most practical situations, managers will not have the benefit of a precise numerical estimate of elasticity based on statistical techniques.

Point versus arc elasticity

We distinguish between point elasticity of demand and arc elasticity of demand on the basis of the size of the change in price and quantity represented by Δ in the preceding elaboration of the elasticity concept. We use point elasticity when the changes in price and quantity are infinitesimally small, since such a small price change represents a virtual point on the demand curve. For more substantial price changes, we speak of arc elasticity, (example above) since we are considering a discrete movement along, or an arc of, the demand curve. For point price elasticity, the formula is modified to read

$$E_P = \frac{dQ_X}{dP_X} \cdot \frac{P_X}{Q_X} \qquad (4)$$

where the letter d is substituted for Δ to reflect the infinitesimally small changes in the variables P_x and Q_x.

Two components of the price elasticity are (a) the inverse of the slope, $\frac{dQ_X}{dP_X}$, which for a linear demand function is constant and (b) $\frac{P_X}{Q_X}$, the coordinates of a point on the demand curve, which is a variable. Therefore, the absolute value of the price elasticity gets larger as we move upward along a given demand curve with the price increasing and the quantity decreasing. Thus, the price elasticity of demand varies along a linear demand curve.

Point price elasticity is the appropriate concept for finding the elasticity value at a particular price level if the demand curve is known. That is, if you already know the slope of the demand curve, you simply weight the reciprocal of that slope by the appropriate price-quantity ratio to find the elasticity.

Demonstration problem

Assume that $Q_X = 10 - 2P_X$, where Q_X is the quantity demanded and P_X is the price of good X. What is the price elasticity at the point on the demand curve where $P_X = \$1$?

Answer:

Plugging in the point elasticity equation we obtain,

$$E_P = -2x\frac{1}{8} = -0.25$$

where $\frac{dQ_X}{dP_X} = -2$. This may be interpreted to mean that when the price is $1, one per cent change in price will cause a 0.25 per cent change in quantity demanded.

Arc price elasticity

This concept is defined as the relative responsiveness of quantity demanded to a discrete change in price, as opposed to an infinitesimal change. As previously discussed, the point formula of Equation (4) reflects a marginal concept, and is valid only for very small movements from point to point along a demand curve. Furthermore, Equation (4) requires that the precise change in Q_X generated by a very small change in P_X be known, which requires that the demand function be known. There are many instances, however, when we are interested in measuring elasticity when the demand function is not known or when our interest lies in a larger segment of the demand curve. For this we need the formula for arc elasticity, which calculates the average elasticity between two points on the demand curve.

Suppose the price of pineapple juice in the local supermarkets is reduced from \$3.50 per litre to \$3.00 per litre, and this causes the average sales of pineapple juice to increase from 200 litres to 300 litres per day. Assuming that all other factors entering the demand function have remained constant, we expect that two price-quantity combinations above are points on the demand curve for the juice.

P_X ($ Per Litre)	**Q_X** (Litre)
\$3.50	200
\$3.00	300

If we use the upper point as our base point, the average (arc) price elasticity between these two points will be

$$elasticity = \frac{\%\Delta Q_X}{\%\Delta P_X} = \frac{\frac{Q_2 - Q_1}{Q_1}}{\frac{P_2 - P_1}{P_1}} = \frac{\frac{300-200}{200}}{\frac{3-3.5}{3.5}} = \frac{\frac{100}{200}}{-\frac{0.5}{3.5}} = -3.5 \qquad (5)$$

If on the other hand the lower point is used as the base point,

$$elasticity = \frac{\frac{Q_1 - Q_2}{Q_2}}{\frac{P_1 - P_2}{P_2}} = \frac{\frac{200-300}{300}}{\frac{3.5-3.0}{3.0}} = \frac{\frac{100}{300}}{-\frac{0.5}{3}} = -2 \qquad (6)$$

As expected, the point price elasticity is lower for the lower point than the higher point on the demand curve. This is because the elasticity at the top of a demand curve is lower than at the bottom of the curve.

Therefore the question is: what to do? Since the arc elasticity is meant to provide an average measure of elasticity over the range rather than at a specific point – either the upper or the lower point – the solution is to change our base for calculating elasticity to an average base (between the two points). These average coordinates designate a point half way between them along a straight-line demand curve.

$$elasticity = \frac{\frac{Q_1 - Q_2}{(Q_1 + Q_2)/2}}{\frac{P_1 - P_2}{(P_1 + P_2)/2}} = \frac{\frac{200 - 300}{(200+300)/2}}{\frac{3.5 - 3.0}{(3.0+3.5)/2}} = \frac{\frac{100}{250}}{-\frac{0.5}{3.25}} = -2.6 \qquad (7)$$

Three points need to be emphasised:

1. The arc elasticity is in fact the point elasticity at the midpoint of the arc, and hence it is a better summary measure of elasticity over the arc than either extreme point.
2. The arc elasticity value becomes increasingly less accurate when we move toward the end of the arc.
3. Finally, the wider the range (arc), the less useful this concept will be.

Demonstration problem

Suppose a demand function is represented by $Q_X = 100 - 10P_X$. What is the price elasticity of demand at the point where the price is \$4? What is the price elasticity at the point where the price is \$5? What is the arc elasticity between these two points?

Answer:

At $P_X = \$4$, $Q_X = 100 - 10\ (4) = 60$, and at $P_X = \$5$, $Q_X = 100 - 10\ (5) = 50$. Hence for the first point (\$4, 60):

$$E_P = \frac{dQ_X}{dP_X} . \frac{P_X}{Q_X} = -10(\frac{4}{60}) = -2/3$$

and for the second point (\$5, 50):

$$E_P = \frac{dQ_X}{dP_X} . \frac{P_X}{Q_X} = -10(\frac{5}{50}) = -1$$

The arc elasticity between these two points is:

$$elasticity = \frac{\frac{Q_1 - Q_2}{(Q_1 + Q_2)/2}}{\frac{P_1 - P_2}{(P_1 + P_2)/2}} = \frac{\frac{50 - 60}{(60+50)/2}}{\frac{5 - 4}{(5+4)/2}} = \frac{\frac{10}{55}}{-\frac{1}{4.5}} = -\frac{45}{55} = -.82$$

Elasticity, marginal revenue and total revenue

Table 3-2 shows the hypothetical prices and quantities demanded of software, the own-price elasticity, and the total revenue (TR = P_xQ_x) for the linear demand function, $Q_X^d = 10 - P_X$. Column 4 and 5, in Table 3-2, show total revenue and marginal revenue. Column 6 shows the price elasticity, $E_P = \frac{dQ_X}{dP_X} . \frac{P_X}{Q_X}$, for a linear demand curve. Given the constant slope, and hence the constant inverse of the slope, for this linear relationship (-1), the absolute value of the price elasticity gets larger as we move upward along a given demand curve with the price increasing and the quantity decreasing. Thus, the price elasticity of demand varies along a linear demand curve.

For points A through E in Table 3-2, an increase in price increases total revenue. Note that we have inverted the demand curve and therefore the inverse demand curve is $P_X^d = 10 - Q_X^d$. For example, an increase in price from $1 to $2 per unit increases total revenue by $7, (MR = $7). Notice that for these two prices, the corresponding elasticity of demand is less than 1 in absolute value.

For points G through K, an increase in price leads to a reduction in total revenue. For example, when the price increases from $7 (where the price elasticity is –2.33) to $8 (where the price elasticity is –4), we see that total revenue decreases by $5 (MR = –$5). The price-quantity combination that maximises total revenue is at point F, where the price elasticity equals one.

Table 3-2 Total revenue, marginal revenue, and elasticity

$P_X^d = 10 - Q_X$

	Price	Quantity	Total Revenue	Marginal Revenue	Price Elasticity
	($/$Q_X$)	(Q_X)	(TR)	(MR)	(E_P)
A	0	10	0	0	0.00
B	1	9	9	9	-0.11
C	2	8	16	7	-0.25
D	3	7	21	5	-0.43
E	4	6	24	3	-0.66
F	5	5	25	1	-1.00
G	6	4	24	-1	-1.50
H	7	3	21	-3	-2.33
I	8	2	16	-5	-4.00
J	1	9	9	-7	-9.00
K	0	10	0	0	-∞

Note that the mathematical expression representing MR function is found as follows:

$$TR = P_X x Q_X = (10 - Q_X)Q_X = 10Q_X - Q_X^2 \text{, and} \tag{8}$$

marginal revenue that is the derivative of total revenue, with respect to Q, is as follows:

$$MR = \frac{dTR}{dQ_X} = 10_X - 2Q_X \tag{9}$$

Note that the marginal revenue equation has the same vertical intercept as the demand equation, but a slope (–2) that is double the slope of the demand curve, (–1).

Price elasticity, total revenue, and marginal revenue are all functionally related. Figure 3-2 shows the demand curve corresponding to the table in the upper panel, while the relationship between price and total revenue is graphed in the lower panel. The upper portion of the demand curve, where the price is higher, the magnitude of price elasticity (in absolute terms) is greater than one, hence the demand is said to be elastic. In the lower portion of the curve, the magnitude of the elasticity is less than one, hence demand is said to be inelastic. At the midpoint, where the price is $5, the magnitude of elasticity is equal to one, and demand is said to be unit elastic.

Figure 3-2 Demand curve corresponding to Table 3-2

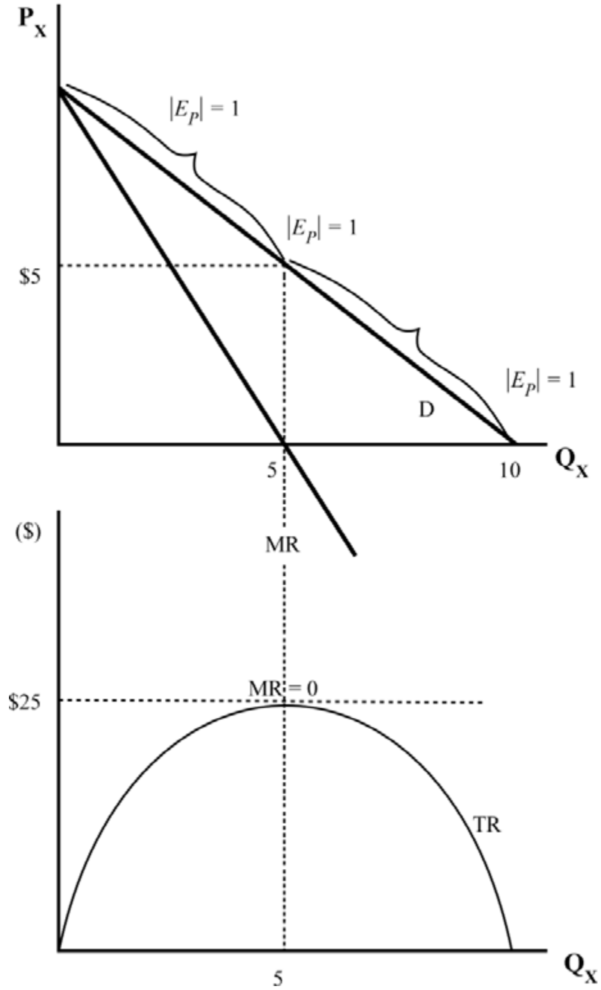


Figure 3-2 shows that when demand is elastic, reducing the price causes total revenue to rise. Why? Because a magnitude of elasticity greater than 1.0 means that if the price is cut by some percentage, the quantity demanded will rise by a greater percentage. The increase in the number of units sold will more than make up for the lower price, and total revenue will increase. By the same reasoning, when demand is inelastic, a price increase will cause total revenue to rise, even though fewer units will be sold. Maximum revenue occurs when the magnitude of elasticity equals 1.0. At this point, any change in price in either direction will cause a reduction in total revenue.

Managers pursuing the ‘cut price and make it up in volume’ strategy are, in effect, using the total revenue test.

The relationship depicted in the demand curve discussed above can be generalised for all linear demand curves. Furthermore, the relationship between the price, marginal revenue, and the price elasticity can be captured by the following simple formula:

$$MR_X = P_X\left(1 - \frac{1}{|E_P|}\right) \quad (10)$$

Factors that determine price elasticity

Four key characteristics of a product influence its elasticity:

1. The degree to which it is viewed as luxury or necessity.
2. The number of substitutes that are available to buyers.
3. The price of the product in relation to buyers' incomes.
4. The amount of time allowed for buyers to react to price changes.

Consider the following examples.

Electricity has few substitutes; therefore the demand for it is inelastic. As more substitutes become available, demand becomes more elastic. AZT, the drug that combats AIDS, at one time had no substitutes; therefore demand was inelastic. The emergence of substitutes will make AZT's demand more elastic.

Suppose the price of salt or pepper doubled. It is such a small portion of expenditures for most people that the price increase would pass almost unnoticed, and quantity demanded would respond only slightly: inelastic demand. In contrast, a doubling in the price of a good that is important in one's budget (petrol, perhaps) will provoke a great response. This is a good example of the role the price plays in the household budget.

The demand for oil offers a good example of short- and long-run elasticity. When OPEC conspired to raise the price of oil in 1973 and again in 1979, consumers, especially in industrialised oil-importing nations, responded by reducing their purchases by a relatively small amount. One economic study at that time pointed out that the short-run elasticity of demand for fuel was about –0.10, a coefficient indicating a very inelastic demand. In the 1980s, however, these consumers had changed their pattern of consumption by car pooling, driving their cars at lower speeds, using more fuel-efficient cars, and turning their thermostats down.

Producers in these countries complemented this response by using more fuel-efficient machinery. Thus the long-run response to increased oil prices was much more elastic than the short-run response.

Demonstration problem

How would you categorise the demand in the market for components and materials by a personal computer manufacturer?

Answer:

This is the market for inputs. Therefore, demand for the input of component and materials is likely to be highly price elastic because consumer demand for personal computers is highly price elastic.

Demonstration problem

How would you categorise the demand for upgrades in the following case? A consumer develops expertise in using a particular word processing package that is incompatible with available alternatives.

Answer:

Since switching costs will be high, the price sensitivity for upgrades will be low.

Elasticity of a product versus elasticity of a brand

Students often mistakenly suppose that just because the demand for a product is inelastic, the demand facing each seller of that product is also inelastic. Consider, for example, wine. Many studies have documented that the demand for wine is price inelastic, with elasticity well below l. This suggests that a general increase in the price of all types of wine from all sources would only modestly affect overall consumption of wine. However, if the price of only one specific winery or vintage of wine increases, the demand for that type would probably drop substantially because consumers would switch to the now lower-priced substitutes. As you might expect, responsiveness is generally greater for a brand than for the product category, for the simple reason that competing brands within the category offer consumers more substitutes.

Brand-level elasticity is higher than industry-level elasticity because consumers can purchase other brands when only one brand raises its price. Brand-level elasticity should also increase as more firms enter the market and more brands are offered.

Demonstration problem

Should a firm use an industry-level elasticity or a brand-1evel elasticity in assessing the impact of a price change?

Answer:

The answer depends on what the firm expects its rivals to do. If a firm expects that rivals will quickly match its price change, then the industry-level elasticity is appropriate. If, by contrast, a firm expects that rivals will not match its price change, then the brand-level elasticity is appropriate.

Other types of elasticity

In addition to price elasticity, there are three other important types of elasticity that economists track: *income elasticity*, *cross-price elasticity*, and *advertising elasticity*. These elasticities measure, respectively, the responsiveness of demand to changes in consumers' income, the price of substitute goods, and advertising.

Income elasticity

You can reasonably expect that when income rises, consumers will buy more of a particular product, less when their income falls. In fact, goods and services that exhibit such a relationship are called *normal.* However, where there is an inverse relationship between changes in income and consumer demand, the products are called *inferior.* Examples of inferior products or services are less-expensive means of transportation (bus versus plane), low quality rice and no-name products. As people's incomes rise, they start to replace these products with their higher-priced substitutes: brand-name products, for example.

Income elasticity is measured in the same way as price elasticity. The percentage change in the quantity demanded is compared with the percentage change in income. The arc income elasticity formula is:

$$E_M = \frac{\%change(quantity)}{\%change(income)} = \frac{\%\Delta Q_X}{\%\Delta M} \qquad (11)$$

where E_M denotes income elasticity. We can categorise the results of this computation as follows:

If the income elasticity coefficient is positive ($E_M > 0$), it indicates that a move in the same direction is occurring with both income and changes in quantity demanded. Products with coefficients greater than zero are called *normal.* As your income increases, you will probably increase your spending on soft drinks, books, clothes, CDs, and so on.

If the income elasticity is negative, it indicates that quantity demanded and level of income move in opposite directions. Therefore, the product is inferior. Potatoes, beans and generic aspirin are good examples of inferior goods. As your income increases, you will probably decrease your spending on such goods (negative elasticity, $E_M < 0$). Conversely, if real income levels decline, the quantity demanded of an inferior good will increase.

If the income elasticity coefficient is greater than one, it indicates that demand is very sensitive to changes in income. In this case, we can refer to the product as a luxury or superior product.

Home ownership might be a luxury. If your income is low, you can only rent. If your income rises, you may qualify for mortgage loans and enter the house-purchasing market. Expenditure on house purchases rises more than the increase in income in such a case.

Alternatively, the point-elasticity formula for income is

$$E_M = \frac{\%\Delta Q_X}{\%\Delta M} = \frac{\Delta Q_X}{\Delta M} . \frac{M}{Q_X} \qquad (12)$$

The implications of income elasticity of demand to the business decision maker are considerable. If the income elasticity for your product exceeds one, the demand for your product will grow more rapidly than does total consumer income. As well, it will fall more rapidly than does total consumer income when income levels are generally falling. Hence, while income elasticity greater than one in a growing economy indicates a growth industry, it also indicates a greater vulnerability to downturns in the level of aggregate economic activity.

Contrastingly, if the income elasticity of demand for your product is positive but less than one, the demand for your product will grow more slowly than the gross national product or consumer income. (However, it will be relatively recession-proof, in the sense that the demand will not react in the volatile fashion of luxury goods.) Third, if your product is regarded as an inferior good by the market as a whole, you must expect the quantity demanded of your product to decline as the gross national product rises.

Therefore, knowledge of a product's income elasticity can help managers in several different ways. First, it can alert them to the impact on demand caused by movements in the macro economy. A recession can be expected to reduce the demand for normal or superior products. In an economic recovery or expansion, these same products should experience rising demand. For example, during the sustained economic expansion of the 1980s, companies that sold luxury consumer products with high-status designer names did very well. In the 1990s, however, many of the same companies experienced sluggish sales because of the slowdown in the economy.

To offset the impact of the business cycle on product demand, a manager might do well to select a portfolio of goods and services with a variety of income elasticity. Thus, in a recession, the demand for a company's inferior or low-income-elasticity products will be sustained and may even increase. In expansionary economic times, the company's high-income-elasticity products would take the lead in sales.

Cross-price elasticity

Cross-price elasticity is a measure of the responsiveness of consumers to changes in the price of a particular good, Good X, relative to changes in the price of substitute or complementary products, Good Y. The cross-elasticity of demand provides a measure of the degree of substitutability, or complementarity, between product X and some other product.

Cross-elasticity of demand is defined as the percentage change in quantity demanded of product X, divided by the percentage change in the price of some product Y.

The arc formula is

$$E_C = \frac{\%\Delta Q_X}{\%\Delta P_Y} \qquad (13)$$

And the point-elasticity formula is

$$E_C = \frac{\Delta Q_X}{\Delta P_Y} \cdot \frac{P_Y}{Q_X} \qquad (14)$$

The main point here is the sign (positive or negative) of the relationship rather than the magnitude. If it is a positive relationship, the goods are substitutes; if it is negative, they are complements. As a secondary issue, the larger (in absolute value) the coefficient, the more related are the two goods. For instance, a small decrease in the price of Pepsi may cause a sizeable decrease in the demand for Coke (close substitutes) but a smaller decrease in the demand for, say, tea.

Knowledge about cross-price elasticity with respect to substitute products is particularly useful to assess the impact on sales of changes in a competitor's prices. For example, what impact will a reduction in the price of Microsoft's Word have on the sales of Word Perfect? To minimise the cross-price elasticity of a product with respect to changes in the price of a substitute, companies spend considerable sums on advertising designed to establish or strengthen brand loyalty. There appears to be increasing cross-price elasticity in consumer goods markets, as evidenced by the growing market share of lower-priced, private-label consumer products. This is becoming quite worrisome for the makers of leading premium brands of consumer products such as Procter & Gamble, Colgate-Palmolive, and Philip Morris.

The cross-price elasticity of a product with respect to complementary products is also important for managers to understand. For example, a seller of computer products can reduce the price of its PCs to stimulate demand for its software. If the profit margin is high for the product whose demand is affected by the cut in the price of the complementary products, this pricing tactic is particularly appealing. For example, a clothing store might reduce the price of its men's suits to stimulate the demand for high-profit margin items such as ties, shirts, and socks. Furthermore, the degree of complementarities between suits and the fashion accessories can be stimulated by the friendly persuasion of the salespersons.

Advertising elasticity

We know that advertising has an impact on the quantity of output sold. Specifically, the quantity demanded of product X will typically show a positive response to the advertising in support of product X, a negative response to the advertising of substitutes, and a positive response to the advertising of complements.

The advertising elasticity of demand for product X measures the responsiveness of the change in quantity demanded to a change in the advertising budget for product X. We expect a positive relationship between advertising and quantity demanded, but we also expect that the responsiveness of sales to advertising will decline as advertising expenditure continues to increase.

Similarly, cross-advertising elasticity of demand measures the responsiveness of quantity demanded of product X to a change in the advertising efforts directed at another product, Y. As stated earlier, one expects cross-advertising elasticity to be negative between substitute products and positive between complementary products. For example, increased advertising efforts for a particular movie would be expected to reduce the quantity demanded of admission tickets to other movies and attractions but to increase the sales at the refreshment kiosk in the lobby of that particular movie theatre. In effect, the increased advertising would have shifted the demand curves to the left for all substitute attractions, while shifting the demand curve to the right for the refreshment kiosk.

$$E_{adv} = \frac{\%\Delta Q_X}{\%\Delta adv} \qquad (15)$$

It is clear that we might calculate the elasticity of demand with respect to any variable that influences the demand for a product.

Obtaining elasticity from the demand function

We will now turn our attention to how to calculate various elasticities from a demand function. In order to do this, we need to recall the demand function. As discussed earlier, a demand function refers to the relationship that exists between the quantity demanded of a particular product and the determinants of demand. In notational shorthand, such is represented as follows:

$$Q_X^d = f(P_X, P_Y, M, adv.,...) \qquad (16)$$

If the demand function is linear, then

$$Q_X^d = a_0 + a_X P_X + a_Y P_Y + a_M M + a_{adv} adv \qquad (17)$$

where a_0, a_X, a_Y, a_M, and a_{adv} are the coefficients of the demand function indicating the marginal effect of each right hand side (independent) variable on the quantity demanded (dependent variable). Note that a_0 shows the marginal effect of other variables not accounted for here.

The relationship between elasticity and these coefficients, as discussed earlier, are summarised as follows:

Own price elasticity: $E_P = a_X \cdot \frac{P_Y}{Q_X}$

Cross price elasticity: $E_C = a_Y \cdot \frac{P_Y}{Q_X}$

Income elasticity: $E_M = a_M \cdot \frac{M}{Q_X}$

Advertising elasticity: $E_{adv} = a_{adv} \cdot \frac{adv}{Q_X}$

Demonstration problem

Suppose the demand function for product X has been estimated to be

$$Q_X^d = 20 + -5P_X + 4P_Y + 0.2M + 3adv\,,$$

where the symbols for the independent variables are the same as those introduced above. Suppose X sells for $20 per unit, the price of alternative product is $30 per unit, average consumer income is $25,000, and the advertising budget is 40 units. Calculate the demand elasticity with respect to all the independent variables. Interpret your results.

Answer:

Plugging in this equation:

$$Q_X^d = 20 - 5(\$20) + 4(\$30) + 0.2(25{,}000) + 3(40) = 5{,}160$$

hence $E_P = -5.\frac{20}{5160} = -0.19$, $E_C = 4.\frac{30}{5160} = 0.23$

$E_M = 0.2.\frac{25{,}000}{5160} = 0.97$, and $E_{adv} = 3\frac{40}{5160} = 0.23$

The following conclusions can be drawn. (a) Since the coefficient of P_y is positive, X and Y are substitutes. (b) The own-price elasticity of demand is inelastic (–0.19). (c) The product under study is almost unit income elastic. And (d) advertising does some good, as indicated by the positive coefficient to adv.

Elasticity for nonlinear demand functions

Managers frequently encounter situations where a product's demand is not a linear function of prices, income, advertising and other demand shifters. In this section we demonstrate that the tools we developed can easily be adapted to these more complex environments.

Suppose the demand function is not a linear function but instead is given by

$$Q_X = a_0.P_X^{a_X}.P_Y^{a_Y}.M^{a_M}.adv^{a_{adv}} \qquad (18)$$

In this case, the variables have a multiplicative impact on $Q_{X,}$ and the coefficients are transformation is used. Expressed in the natural logarithm (log):

$$\ln Q_X = \ln a_0 + a_X.\ln P_X + a_Y.\ln P_Y + a_M.\ln M + a_{adv}.\ln adv \qquad (19)$$

When the demand function is log-linear, as in equation (19), the coefficients are the elasticity.

Demonstration problem

Suppose the demand for product X is given in log-linear as follows: $\ln Q_X = 50 - 0.75 \ln P_X + 1.2 \ln P_Y + 2 \ln M$. What are the demand elasticity of product X with respect to income and its own price?

Answer:

$$E_P = -0.75$$
$$E_M = 1.2.$$

Module summary

Summary

The demand for a product that a firm faces depends on the price of the product, consumers' income, the price of related products, advertising and promotions. Such a relationship is referred to as a demand function. Elasticity is the most commonly used measurement of the sensitivity of demand to any of its determinants. In general, elasticity is defined as a percentage change in quantity (the dependent variable) of output demanded caused by a one percentage change in an independent variable. Elasticity is a general concept that applies to any function, price of the product, income, price of related products, advertising, etc. The demand function is unit elastic at the point where marginal revenue is zero and total revenue is at a maximum. Below this point, demand is inelastic, marginal revenue is negative, and an increase in price increases total revenue. Above this point, demand is elastic and the firm can increase total revenue by reducing price.

Assignment

Assignment

1. For the demand function Q = 10 – 0.5P
    a. Find the point elasticity of demand. Briefly explain why the value is negative.
    b. Find the point elasticity of demand at P = 5.
2. For the inverse demand function P = 5 – 2Q
    a. Find the revenue function (i.e., revenue as a function of quantity).
    b. Find the elasticity of demand.
    c. At which point the elasticity of demand is one? What will be the effect on revenue if you change the price by 1 per cent at this point? Explain.

Assessment

Assessment

1. The cross-price elasticity of tea and coffee is estimated to be 0.15. Explain why the value is positive. If the coffee price decreases by 25 per cent, what will be the effect on sales of tea?
2. For the demand function $Q = 10P^{-0.5}$
   a. Find the elasticity of demand.
   b. Find the marginal revenue function.
   c. Find the relationship between marginal revenue and elasticity of demand. Briefly explain.
3. Find the income elasticity for a good with the demand function $Q = 10MP^{-1}$, at M = 50,000 and P = 100. (Here, Q = Quantity, M = income, and P = price). Is this good normal or inferior? Explain.
4. The demand for yogurts of brand 'A' is estimated to be, $Q = 50 – 5P_a + 4P_b + 0.05M$ Q: Quantity, P_a: Price of brand 'A', P_b: Price of brand 'B', M = Income
   a. Find the own price elasticity of demand.
   b. Find the cross price elasticity of demand.
   c. Find the income elasticity of demand.
   d. Are 'A' and 'B' substitutes or complements? Explain.
   e. Is 'A' an inferior good? Explain why or why not.

Assessment answers

1. The cross price elasticity of tea and coffee is 0.15. The positive value says that if the price of coffee goes up, then the quantity demanded of tea will also go up. Thus tea and coffee are substitutes.

   $$\%\Delta Q_t \,/\, \%\Delta P_t = 0.15 \Rightarrow \%\Delta Q_t \,/\, 25 = 0.15 \Rightarrow \%\Delta Q_t = 3.75$$

   Quantity demanded of tea will increase by 3.75%

2. $Q = 10P^{-0.5}$…… $P = 100Q^{-2}$

   Elasticity = $P/Q \times dQ/dP = P/Q \times 10(-0.5)P^{-1.5} = (-0.5)(10P^{-0.5})/Q = -0.5$

   Revenue function $R(Q) = PQ = 100Q^{-2} \times Q = 100/Q$

   Marginal Revenue = $dR/dQ = -100Q^{-2}$

   Marginal Revenue = $dR/dQ = d(PQ)/dQ = P + Q_x$ dip/dQ = $P[1+Q/P \times dP/dQ] = P[1+ 1/(P/Q \times dQ/dP)] = P[1 + 1/E_d]$, where E_d = elasticity of demand.

3. $Q = 10MP^{-1}$

   ln Q = ln 10 + ln M – ln P

   d lnQ/d ln M = 1 = income elasticity of demand

   With a positive change in income, the quantity demanded for the good will also change positively. Therefore, the good is normal. Therefore, at M = 50,000 and P = 100, the elasticity will be 1.

4. $E(Q,P_a) = -5 \times P_a/Q$

   $P_a = 50$, $P_b = 50$, M = 10,000. Plugging in, Q = 50 – 5(50) + 4(50) + 0.05(10,000) = 500

   $E(Q,P_a) = -5.\ 50/500 = -0.5$

   $E(Q,P_b) = +4 \times P_b/Q = 0.4$

   $E(Q,M) = 0.05 \times M/Q = 1$

   $E(Q,P_b) > 0$, therefore, A and B are substitutes.

   $E(Q,M) > 1$, therefore A is a normal good.

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