Permutation and Combination

4

The aim of this unit is to help the learners to learn the concepts of permutation and combination. It deals with nature of permutation and combinations, basic rules of permutations and combinations, some important deduction of permutations and combinations and its application followed by examples.

Blank Page

Lesson-1: Permutation

After studying this lesson, you should be able to:

- Discuss the nature of permutations;
- Identify some important deduction of permutations;
- Explain the fundamental principles and rules of permutations;
- Highlight on some model application of permutations;

Definition of Permutation

Permutations refer to different arrangements of things from a given lot taken one or more at a time. The number of different arrangements of r things taken out of n dissimilar things is denoted by ^nP_r.

Permutations refer to different arrange-ments of things from a given lot taken one or more at a time.

For example, suppose there are three items *x, y* and *z*.

The different arrangements of these three items taking 2 items at a time are: *xy, yx, yz, zy, zx* and *xz*. Thus $^nP_r = {}^3P_2 = 6$.

Again all the arrangements of these three items taking 3 items at a time are: *xyz, xzy, yzx, yxz, zxy* and *zyx*. Thus $^nP_r = {}^3P_3 = 6$.

Hence it is clear that the number of permutations of 3 things by taking 2 or 3 items at a time is 6.

Fundamental Principles of Permutation

If one operation can be done in m different ways where it has been done in any one of these ways, and if a second operation can be done in n different ways, then the two operations together can be done in $(m\times n)$ ways.

Permutations of Things All Different

Permutations of *'n'* different things taken *'r'* at a time is denoted by ^nP_r where $r \leq n$. Here, $^nP_r = n.(n-1).(n-2)........ (n-r+1)$.

Permutations of 'n' different things taken 'r' at a time is denoted by ^nP_r.

Therefore, the first place can be filled up in n ways.

The first two places can be filled up in $n.(n-1)$ ways.

The first three places can be filled up in $n.(n-1).(n-2)$ ways.

Permutation of Things Not All Different

The number of permutation of *'n'* things taken *'r'* at a time in which k_1 elements are of one kind, k_2 elements are of a second kind, k_3 elements are of a third kind and all the rest are different is given by:

$$^nP_r = \frac{r!}{K_1!.K_2!.K_3!......K_n!}$$

Circular Permutations

The number of distinct permutations of n objects taken n at a time on a circle is (n –1)!. In considering the arrangement of keys on a chain or

The number of distinct permutations of n objects taken n at a time on a circle is (n –1)!

beads on a necklace, two permutations are considered the same if one is obtained from the other by turning the chain or necklace over. In that case there will be $\frac{1}{2}(n-1)!$ ways of arranging the objects.

Some Important Deduction of Permutations

(i) $^{n}P_{n} = n.(n-1).(n-2)$............... to n factors

$$= n.(n-1).(n-2)\ldots\ldots\ldots\ldots \{n-(n-1)\}$$

$$= n.(n-1).(n-2)\ldots\ldots\ldots\ldots 1$$

$$= n.(n-1).(n-2)\ldots\ldots\ldots\ldots 3.2.1.$$

$$= n!$$

(ii) $^{n}P_{n-1} = \frac{n!}{\{n-(n-1)\}!}$ [since, $^{n}P_{r} = \frac{n!}{(n-r)!}$]

$$= \frac{n!}{\{n-n+1)\}!} = \frac{n!}{1!} = n!$$

(iii) $^{n}P_{r} = n.\ ^{n-1}P_{r-1}$

or, $\frac{n!}{(n-r)!} = n.\ \frac{(n-1)!}{\{(n-1)-(r-1)\}!}$

or, $\frac{n!}{(n-r)!} = n.\ \frac{(n-1)!}{(n-r)!}$

or, $\frac{n!}{(n-r)!} = \frac{n!}{(n-r)!}$ [since, $n\ (n-1)! = n!$]

$\therefore\ ^{n}P_{r} = n.\ ^{n-1}P_{r-1}$

(iv) $^{n}P_{r} = n.(n-1).(n-2)\ldots\ldots\ldots (n-r+1)$

$$= \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

(v) $^{n}P_{r} = {}^{n-1}P_{r} + r.\ ^{n-1}P_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!} + r.\ \frac{(n-1)!}{\{(n-1)-(r-1)\}!}$$

$$= \frac{(n-1)!}{(n-1-r)!} + \frac{r.(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r.(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[1+\frac{r}{(n-r)}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!} \times \left[\frac{n-r+r}{(n-r)}\right]$$

$$= \frac{n(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{n!}{(n-r)!} = {}^{n}P_{r}$$

$$\therefore\ {}^{n-1}P_{r} + r.\ {}^{n-1}P_{r-1} = {}^{n}P_{r}$$

The following examples contain some model application of permutations.

Example-1:

A store has 8 regular door ways and 5 emergency doors which can be opened only from the inside. In how many ways can a person enter and leave the store?

Solution:

To enter the store, a person may choose any one of 8 different doors. Once inside he may leave by any one of (8+5) =13 doors.

$\therefore$ The total number of different ways is (8 × 13) =104.

Example-2:

There are 10 routes for going from a place Chittagong to another place Dhaka and 12 routes for going from Dhaka to a place Khulna. In how many ways can a person go from Chittagong to Khulna Via Dhaka?

Solution:

There are 10 different routes from Chittagong to another place Dhaka, the person can finish the first part of the journey in 10 different ways. And when he has done so in any one way, he will get 12 different ways to finish the second part. Thus one way of going from Chittagong to Dhaka gives rise to 12 different ways of completing the journey from Chittagong to Khulna via Dhaka.

Hence the total number of different ways of finishing both the parts of the journey as desired = (No. of ways for the 1st part × No. of ways for 2nd part) = (10 × 12) = 120.

Example-3:

There are 8 men who are to be appointed as General Manager at 8 branches of a supermarket chain. In how many ways can the 8 men be assigned to the 8 branches?

Solution:

Since every re-arrangement of the 8 men will be considered as a different assignment, the number of ways will be

$$^{8}P_{8} = \frac{8!}{(8-8)!} = (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) = 40{,}320 \text{ ways}$$

Example-4:

Six officials of a company are to fly to a conference in Dhaka. Company policy states that no two can fly on the same plane. If there are 9 flights available, how many flight schedules can be established?

Solution:

The number of flight schedule can be established for the six officials in

$$^{9}P_{6} = \frac{9!}{(9-6)!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = 60480 \text{ ways}$$

Thus the total number of ways is 60480.

Example-5:

In how many ways can 3 boys and 5 girls be arranged in a row so that all the 3 boys are together?

Solution:

The 3 boys will always be kept together, so we count the 3 boys as one boy. As a result the number of persons involved to be arranged in a row is 6.

They can be arranged in 6! ways = (6 × 5 × 4 × 3 × 2 × 1) = 720 ways.

But these 3 boys themselves can be arranged in 3! ways, i.e. (3 × 2 × 1) = 6 ways.

Hence the required number of arrangement in which the boys are together will be,

= (720 × 6) = 4320 ways.

Example-6:

Out of the letters *P, Q, R, x, y* and *z*, how many arrangements can be made (i) beginning with a capital; (ii) beginning and ending with a capital.

Solution:

(i) One capital letter out of given 3 capital letters can be chosen in ${}^{3}P_{1}$ = 3 ways. Remaining the other five letters can be arranged among themselves in 5! ways, i.e. in (5 × 4 × 3 × 2 × 1) = 120 ways.

Hence the total number of arrangements beginning with a capital = (120 × 3) = 360.

(ii) Two capital letters out of given 3 capital letters can be chosen in ${}^{3}P_{2}$ = 6 ways. For each choice of these two letters, remaining four letters can be arranged in 4! ways, i.e. in (4 × 3 × 2 × 1) = 24 ways.

Therefore the required number of arrangements beginning and ending with a capital

= (6 × 24) = 144.

Example-7:

Six papers are set in an examination of which two are mathematical. In how many different orders can the papers be arranged so that (i) the two mathematical papers are together; (ii) the two mathematical papers are not consecutive.

Solution:

(i) We count the two mathematical papers as one, so that the total number of arrangement can be done in 5! ways, i.e., in (5 × 4 × 3 × 2 × 1) = 120 ways.

Two mathematical papers can be arranged within themselves in 2! = (2 × 1) = 2 ways.

Hence the required number of arrangement in which the mathematical papers are always together is = (120 × 2) = 240.

(ii) Again the total number of possible arrangements is 6! = (6×5×4×3×2×1) = 720 ways.

Hence the total number of arrangements in which mathematical papers are not consecutive is = (720 – 240) = 480 ways.

Example-8:

How many different numbers of 3 digits can be formed from the digits 1, 2, 3, 4, 5 and 6, if digits are not repeated? What will happen if repetitions are allowed?

Solution:

If the repetition of digits is not allowed then the required number of arrangements is, ${}^{6}P_{3} = \dfrac{6!}{(6-3)!} = \dfrac{6\times5\times4\times3!}{3!}$ = 120 ways.

If the repetition of digits are allowed then the required number of arrangements is,

$= (n \times n \times n) = (6 \times 6 \times 6) = 216$ ways.

Therefore 120 and 216 different numbers can be formed respectively by repeating and not repeating digits 1, 2, 3, 4, 5 and 6.

Example-9:

How many words can be formed with the help of 3 consonants and 2 vowels, such that no two consonants are adjacent?

Solution:

Let B, C and D are three consonants and A, E are two vowels. According to the question it is represented in the following figure.

B, A, C, E, D,

The vowels A and E occupy the two positions between B, C and C, D. Each of such arrangements of consonants gives rise to two arrangements of vowels. But 3 consonants can be arranged in 3 places in 3! ways, i.e., $(3 \times 2 \times 1) = 6$ ways.

Hence the total number of arrangement is $= (2 \times 6) = 12$.

Thus the number of different words to be formed is 12.

Example-10:

How many different words can be made out of the letters of the word 'ALLAHABAD'? In how many of these with the vowels occupy the even places?

Solution:

The word 'ALLAHABAD' has 9 letters, of which 'A' occurs four times, L occurs twice and the rest all are different.

Hence the required number of permutations is,

$$= \frac{9!}{4!2!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2 \times 1} = 7560$$

The word ALLAHABAD consists of 9 letters. There are 4 even place which can be filled up by the 4 vowels in 1 way only, since all the vowels are similar (all are 'A's). Moreover the remaining 5 places can be filled up by the 5 consonants of which 2 are similar in $= \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = 60$ ways.

Hence the required number of arrangement is $(1 \times 60) = 60$.

Example-11:

In how many ways can 5 boys and 5 girls, sit at a round table so that no 2 boys are together.

Solution:

Suppose that the girls be seated first. They can sit in $(5 - 1)! = 4!$ ways, i.e., in $(4 \times 3 \times 2 \times 1) = 24$ ways.

Now since the places for the boys in between girls are fixed, the option is there for the boys to occupy the remaining 5 places. There are 5! ways, i.e., $(5 \times 4 \times 3 \times 2 \times 1) = 120$ ways for the boys to fill up the 5 places in between 5 girls seated around a table already.

Therefore the total numbers of arrangement in which both girls and boys can be seated are, $(24 \times 120) = 2880$ ways.

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Indicate how many 4 digit numbers smaller than 6,000 can be formed from the digits 2, 4, 5, 6, 8, 9?
2. Find the number of arrangements than can be made out of the letters of the word "ASSASSINATION".
3. Indicate how many 5 digit numbers can be formed from the digits 2, 3, 5, 6, 8, 9 where 6 and 9 must be included in all cases.
4. In how many ways can 6 persons formed a ring?
5. How many different arrangements can be made of all the letters of the word "ACCOUNTANTS"? In how many of them the vowels stand together?
6. In how many ways 3 boys and 5 girls be arranged in a row so that all the 5 girls, are together?
7. In how many ways can the letters of the word "EQUATION" be arranged so that the consonants may occupy only odd positions?
8. In how many ways can seven supervisors and six engineers sit for a round table discussion so that no two supervisors are setting together?
9. Find the number of permutations of the word ENGINEERING.

Lesson-2: Combinations

After studying this lesson, you should be able to:

- State the nature of combinations;
- Explain the important deductions of combinations;
- Highlight on some model applications of combinations.

Definition of Combination

bination refers to erent set of groups le out of a given without repeating lement, taking one nore of them at a

Combination refers to different set of groups made out of a given lot, without repeating an element, taking one or more of them at a time. In other words, each of the groups which can be formed out of *n* things taking *r* at a time without regarding the order of things in each group is termed as combination. It is denoted by ${}^nC_r$.

For example, suppose there are three things *x, y* and *z*.

The combinations of 3 things taken 2 things at a time are: *xy, yz, zx*

Thus ${}^nC_r = {}^3C_2 = 3$.

Some Important Deductions of Combinations

(i) $${}^nC_r = \frac{n!}{r!(n-r)!}$$

Generally ${}^nC_r$ combinations would produce $({}^nC_r \times r!)$ permutations; i.e., $({}^nC_r \times r!) = {}^nP_r$.

Hence, $({}^nC_r \times r!) = {}^nP_r$

$${}^nC_r = \frac{{}^nP_r}{r!} = \frac{n.(n-1).(n-2)..................(n-r+1)}{r!}$$

$${}^nC_r = \frac{n.(n-1).(n-2)..................(n-r+1).(n-r)!}{r.(n-r)!}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

(ii) $${}^nC_0 = \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1 \qquad \text{[since } 0! = 1\text{]}$$

(iii) $${}^nC_1 = \frac{n!}{1!(n-1)!} = \frac{n.(n-1)!}{1!(n-1)!} = n$$

(iv) $${}^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!} = 1$$

(v) $${}^nC_{n-1} = \frac{n!}{(n-1)!\{n-(n-1)\}!} = \frac{n.(n-1)!}{(n-1)!(n-n+1)!} = n$$

$$\therefore {}^nC_1 = {}^nC_{n-1}$$

(vi) $^nC_r = {^nC_{n-r}}$

$$= \frac{n!}{(n-r)!\{n-(n-r)\}!} = \frac{n!}{(n-r)!\,r!} = {^nC_r}$$

Therefore, $^nC_r = {^nC_{n-r}}$

(vii) Prove that $^{n+1}C_r = {^nC_r} + {^nC_{r-1}}$

We know that $^nC_r = \frac{n!}{r!(n-r)!}$

$$\therefore {^nC_r} + {^nC_{r-1}} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\{(n-(r-1)\}!}$$

$$= \frac{n!}{r.(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r}+\frac{1}{(n-r+1)}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r.(n-r+1)}\right]$$

$$= \frac{(n+1).n!}{r.(r-1)!.(n-r+1).(n-r)!}$$

$$= \frac{(n+1)!}{r!.(n-r+1)!}$$

$$= \frac{(n+1)!}{r!.\{(n+1)-r\}!} = {^{n+1}C_r} \text{ (Proved).}$$

The following examples illustrate some model applications of combinations.

Example-1:

Find out the number of ways in which a cricket team consisting of 11 players can be selected from 14 players. Also find out how many of these ways (i) will include captain (ii) will not include captain?

Solution:

The numbers of ways in which 11 out of 14 players can be selected are

$$^nC_r = {^{14}C_{11}} = \frac{14!}{11!(14-11)!} = \frac{14\times 13\times 12\times 11!}{11!\times 3\times 2\times 1} = 364$$

(i) As captain is to be kept in every combination, we are to choose 10 out of the remaining 13 players. Therefore the required number of ways,

$$^{13}C_{10} = \frac{13!}{10!(13-10)!} = \frac{13\times 12\times 11\times 10!}{10!\times 3\times 2\times 1} = 286 \text{ ways}$$

(ii) In this case as captain is to be excluded, therefore, we are to choose 11 out of remaining 13 players which can be done in,

$$^{13}C_{11} = \frac{13!}{11!(13-11)!} = \frac{13\times 12\times 11!}{11!\times 2\times 1} = 78 \text{ ways.}$$

Example-2:

Out of 17 consonants and 5 vowels, how many different words can be formed each containing 3 consonants and 2 vowels?

Solution:

3 consonants can be selected out of 17 in $^{17}C_3$ ways and 2 vowels can be selected out of 5 in $^{5}C_2$ ways.

$\therefore$ The number of selections having 3 consonants and 2 vowels = $^{17}C_3 \times {}^{5}C_2$ ways.

Each of these selections contains 5 letters which can be arranged among themselves in 5! ways. Therefore the total number of words = $^{17}C_3 \times {}^{5}C_2 \times 5!$

$$= \frac{17!}{3!(17-3)!} \times \frac{5!}{2!(5-2)!} \times 5!$$

$$= \frac{17\times 16\times 15\times 14!}{3\times 2\times 1\times 14!} \times \frac{5\times 4\times 3!}{2\times 1\times 3!} \times 5.4.3.2.1 = 8{,}16{,}000.$$

Example-3:

From 6 boys and 4 girls, a committee of 6 is to be formed. In how many ways can this be done if the committee contains (i) exactly 2 girls, or (ii) at least 2 girls?

Solution:

(i) The committee of 6 is to contain 2 girls and 4 boys.

Therefore 2 girls can be selected out of 4 girls in $^{4}C_2 = \frac{4.3.2!}{2.1.2!} = 6$ ways.

The remaining 4 boys can be selected out of 6 in $^{6}C_4 = \frac{6.5.4!}{4!.2.1} = 15$ ways

Therefore the required number of ways = (6 × 15) = 90 ways.

(ii) In this case the committee of 6 can be formed in the following ways.

(a) 2 girls and 4 boys, (b) 3 girls and 3 boys and (c) 4 girls and 2 boys.

We now consider all these 3 cases:

In case of (a) the committee of 6 can be formed as explained above in $^{4}C_2 \times {}^{6}C_4$ ways.

Accordingly there are $^{4}C_3 \times {}^{6}C_3$ and $^{4}C_4 \times {}^{6}C_2$ ways of forming the committee in cases of (b) and (c) respectively.

Hence, the total number of different ways

$= ({}^4C_2 \times {}^6C_4) + ({}^4C_3 \times {}^6C_3) + ({}^4C_4 \times {}^6C_2)$

$= (\frac{4!}{2!(4-2)!} \times \frac{6!}{4!(6-4)!}) + (\frac{4!}{3!(4-3)!} \times \frac{6!}{3!(6-3)!}) + (\frac{4!}{4!(4-4)!} \times \frac{6!}{2!(6-2)!})$

$= [(6 \times 15) + (4 \times 20) + (1 \times 15)] = (90 + 80 + 15) = 185.$

Example-4:

In an examination, a candidate is required to answer 6 out of 12 questions which are divided into two groups each containing 6 questions and he is not permitted to attempt more than 4 questions from each group. In how many ways can he make up his choice?

Solution:

The candidate can have following three choices:

(i) 2 questions from 1st group and 4 questions from 2nd group.

(ii) 3 questions from 1st group and 3 questions from 2nd group.

(iii) 4 questions from 1st group and 2 question from 2nd group.

Now first choice can be made up in ${}^6C_2 \times {}^6C_4$ ways, second choice can be made up in ${}^6C_3 \times {}^6C_3$ ways, and third choice can be made up in ${}^6C_4 \times {}^6C_2$ ways.

Hence the total number of ways that he can make up his choice

$= ({}^6C_2 \times {}^6C_4) + ({}^6C_3 \times {}^6C_3) + ({}^6C_4 \times {}^6C_2)$

$= (\frac{6!}{2!(6-2)!} \times \frac{6!}{4!(6-4)!}) + (\frac{6!}{3!(6-3)!} \times \frac{6!}{3!(6-3)!}) + (\frac{6!}{4!(6-4)!} \times \frac{6!}{2!(6-2)!})$

$= [(15 \times 15) + (20 \times 20) + (15 \times 15)] = (225 + 400 + 225) = 850.$

Example-5:

The question paper of admission test in 1st years B.B.A (Hons) course in Chittagong University contains 20 questions divided into 4 groups of five questions each. In how many ways can an examinee answer 10 questions taking at least 2 questions from each group?

Solution:

The questions may be answered in the following ways:

	1st group (5)	2nd group (5)	3rd group (5)	4th group (5)
(a)	2	2	2	4
(b)	2	2	4	2
(c)	2	4	2	2
(d)	4	2	2	2
(e)	3	3	2	2

	1st group (5)	2nd group (5)	3rd group (5)	4th group (5)
(f)	3	2	3	2
(g)	2	2	3	3
(h)	2	3	2	3
(i)	2	3	3	2
(j)	3	2	2	3

For (a), the total number of ways of selecting questions from the groups is

$$= {}^5C_2 \times {}^5C_2 \times {}^5C_2 \times {}^5C_4 = 5000$$

Similarly,

For (b) $= {}^5C_2 \times {}^5C_2 \times {}^5C_4 \times {}^5C_2 = 5000$

For (c) $= {}^5C_2 \times {}^5C_4 \times {}^5C_2 \times {}^5C_2 = 5000$

For (d) $= {}^5C_4 \times {}^5C_2 \times {}^5C_2 \times {}^5C_2 = 5000$

For (e) $= {}^5C_3 \times {}^5C_3 \times {}^5C_2 \times {}^5C_2 = 10000$

For (f) $= {}^5C_3 \times {}^5C_2 \times {}^5C_3 \times {}^5C_2 = 10000$

For (g) $= {}^5C_2 \times {}^5C_2 \times {}^5C_3 \times {}^5C_3 = 10000$

For (h) $= {}^5C_2 \times {}^5C_3 \times {}^5C_2 \times {}^5C_3 = 10000$

For (i) $= {}^5C_2 \times {}^5C_3 \times {}^5C_3 \times {}^5C_2 = 10000$

For (j) $= {}^5C_3 \times {}^5C_2 \times {}^5C_2 \times {}^5C_3 = 10000$

Hence the total number of ways

$$= (5000 + 5000 + 5000 + 5000 + 10000 + 10000 + 10000 + 10000 + 10000 + 10000)$$

$$= 80{,}000.$$

Example-6:

A cricket team consisting of 11 players is to be formed from 16 players of whom 4 can be bowlers and 2 can keep wicket and the rest can neither be bowler nor keep wicket. In how many different ways can a team be formed so that the teams contain (i) exactly 3 bowlers and 1 wicket keeper, (ii) at least 3 bowlers and at least 1 wicket keeper?

Solution:

(i) A cricket team of 11 is to be formed with exactly 3 bowlers and 1 wicket keeper.

3 bowlers can be selected out of 4 in ${}^{4}C_{3}$ ways, 1 wicket keeper can be selected out of 2 in ${}^{2}C_{1}$ ways and the other 7 players can be selected from the remaining 10 players in ${}^{10}C_{7}$ ways.

Hence the total number of ways in which the cricket team can be formed

$$= {}^{4}C_{3} \times {}^{2}C_{1} \times {}^{10}C_{7}$$

$$= \frac{4!}{3!(4-3)!} \times \frac{2!}{1!(2-1)!} \times \frac{10!}{7!(10-7)!}$$

$$= (4 \times 2 \times 120) = 960.$$

(ii) Since at least 3 bowlers and at least one wicket keeper is to be included in the cricket team of 11 players, the team can be formed by choosing.

(a) 3 bowlers, 1 wicket keeper and 7 other players.

(b) 3 bowlers, 2 wicket keeper and 6 other players.

(c) 4 bowlers, 1 wicket keeper and 6 other players.

(d) 4 bowlers, 2 wicket keeper and 5 other players.

We now consider all these 4 cases.

(a) 3 bowlers, 1 wicket keeper and 7 other players can be selected in

$$= {}^{4}C_{3} \times {}^{2}C_{1} \times {}^{10}C_{7} = \frac{4!}{3!(4-3)!} \times \frac{2!}{1!(2-1)!} \times \frac{10!}{7!(10-7)!}$$

$$= (4 \times 2 \times 120) = 960 \text{ ways.}$$

(b) 3 bowlers, 2 wicket keeper and 6 other players can be selected in

$$= {}^{4}C_{3} \times {}^{2}C_{2} \times {}^{10}C_{6} = (4 \times 1 \times 210) = 840 \text{ ways.}$$

(c) 4 bowlers, 1 wicket keeper and 6 other players can be selected in

$$= {}^{4}C_{4} \times {}^{2}C_{1} \times {}^{10}C_{6} = (1 \times 2 \times 210) = 420 \text{ ways}$$

(d) 4 bowlers, 2 wicket keeper and 5 other players can be selected in

$$= {}^{4}C_{4} \times {}^{2}C_{2} \times {}^{10}C_{5} = (1 \times 1 \times 252) = 252 \text{ ways}$$

Therefore, the total number of ways

$$= (960 + 840 + 420 + 252) = 2472.$$

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. A committee of 5 is to be formed from 14 students. How many different ways this can be done so as always to (i) include 2 particular students; and (ii) exclude 3 particular students?
2. A question paper contains six questions, each having an alternative. In how many ways can an examinee answer one or more questions?
3. A committee consists of 5 members is to be formed out of 6 men and 4 women. How many types of committees can be formed so that at least 2 women are always there?
4. Out of 10 consonants and 4 vowels, how many words can be formed each containing 3 consonants and 2 vowels?
5. From 6 boys and 4 girls, 5 are to be selected for admission into a particular course. In how many ways can this be done if there must be exactly 2 girls?
6. In how many ways a committee of 5 members can be formed out of 8 professors? How often will each professor be selected? If one particular professor is always included, what will be the number of ways? In how many ways the committee can be formed if one particular professor is always excluded?